



# Using the Pythagorean Theorem to Explore and Measure Topography in 2D/3D Space

Lesson plan and more resources are  
available at: [aka.ms/hackingstem](https://aka.ms/hackingstem)



# Hacking STEM

Hacking STEM is a free resource, delivering inquiry and project-based lessons that complement current STEM curriculum. In this project we use measuring tools to explore distance, angles and the Pythagorean theorem.

## Exploring and Measuring Topography in 2D/3D Space

Students build measuring tools from cardboard or LEGO® bricks to create an Initial Transportation Plan for an island national park in Excel. Next, they use the Pythagorean Theorem to design their road and bring their national park to life by adding topographic elements in 3D paint.

### Contents

03	Activity Overview
04	Intro to Pythagorean theorem
05	<b>Part 1: Plan and Build</b>
06	Things You'll Need
07	Construct the Map & Tools
16	<b>Part 2: Connect Your Tools</b>
17	Things You'll Need
19	Connect to Arduino
20	Flash the Arduino
21	Download Project Cordoba
22	<b>Part 3: Visualize the Data</b>
23	Things You'll Need
32	Templates



## Lesson Plan Notebooks

Contains lessons, materials, and activities to support teaching this unit. Mapped to the NGSS and ISTE standards.

Go to:  
[aka.ms/measuringtoolslessonplan](https://aka.ms/measuringtoolslessonplan)  
to access the OneNote notebook and other resources.



# Activity Overview

In this project, students use the Pythagorean theorem to explore and measure topography in 2D/3D space. Students take on the role of environmental engineers and surveyors who have been tasked with creating an Initial Transportation Plan for a new national park in a preserved chain of islands.

## Plan and Build

Students start their exploration of topography by examining a digital proof of the Pythagorean theorem in Excel. Next, they develop a proposed transportation plan on paper connecting important aspects of park's features and assets. They then use the concepts learned from the proof to calculate each road segment distance and build tools that can be used to digitize distances and angles for entry into Excel.

## Connect Your Tools

The student built tools for measuring distance and angles are made of cardboard or LEGO® bricks and a digital rotary encoder. They are connected to Excel via an Arduino microcontroller and are used to transfer points from a student created map. Students need to measure precise distance and angle measurements in order to accurately translate the points from their physical two-dimensional map into a digital two-dimensional and three-dimensional representation of their national park in Excel and Paint 3D.

## Visualize the Data

This lesson uses a custom Excel workbook that includes a tool calibration worksheet plus two additional worksheets for investigating and visualizing live data. The first worksheet allows students to use their Angle Finder and Distance Wheel tool to transfer data into Excel and recreate their two-dimensional map digitally within the worksheet. The second worksheet allows the students to start thinking about height and slope of mountain passes that are created in the national park. With this, students can then visualize and calculate the height of the mountains using the Pythagorean theorem. Once the lesson is completed, students are encouraged to export their digital two-dimensional maps into Paint 3D and begin designing in a three-dimensional environment.

**21st century technical skills explored in this activity include:**



Mechanical Engineering



Electrical Engineering



Software Engineering



Data Science



### Hack our projects

We love innovation and encourage you to hack our activities and make them your own.



### Steps for success

We've indicated important steps with a rocket ship. Use extra care and precision when you see the rocket to ensure great results!



### Substituting everyday objects

Similar items can be substituted for most materials according to availability.



### Sourcing specialized materials

You can find an online shopping list for this entire lesson at:

[aka.ms/measuringtoolsshoppinglist](https://aka.ms/measuringtoolsshoppinglist)

# Intro to Pythagorean Theorem

Go to [aka.ms/measuringtoolsworkbook](https://aka.ms/measuringtoolsworkbook) to download the Intro to Pythagorean theorem Excel workbook. Once it is downloaded, open the file and it should look like the photo below:

The screenshot shows an Excel interface with two worksheets. The left worksheet, titled "Pythagorean theorem", contains a right-angled triangle with a height of 3 and a base of 4, and a hypotenuse of 5. It includes a theorem statement  $a^2 + b^2 = c^2$  and a grid-based visualization of the squares. The right worksheet, titled "Explore the Pythagorean Theorem", features sliders for "a" and "b" and a corresponding grid visualization. It shows the equation  $a^2 + b^2 = c^2$  with numerical values:  $3^2 + 4^2 = c^2$ ,  $9 + 16 = c^2$ ,  $25 = c^2$ , and finally  $5 = c$ .

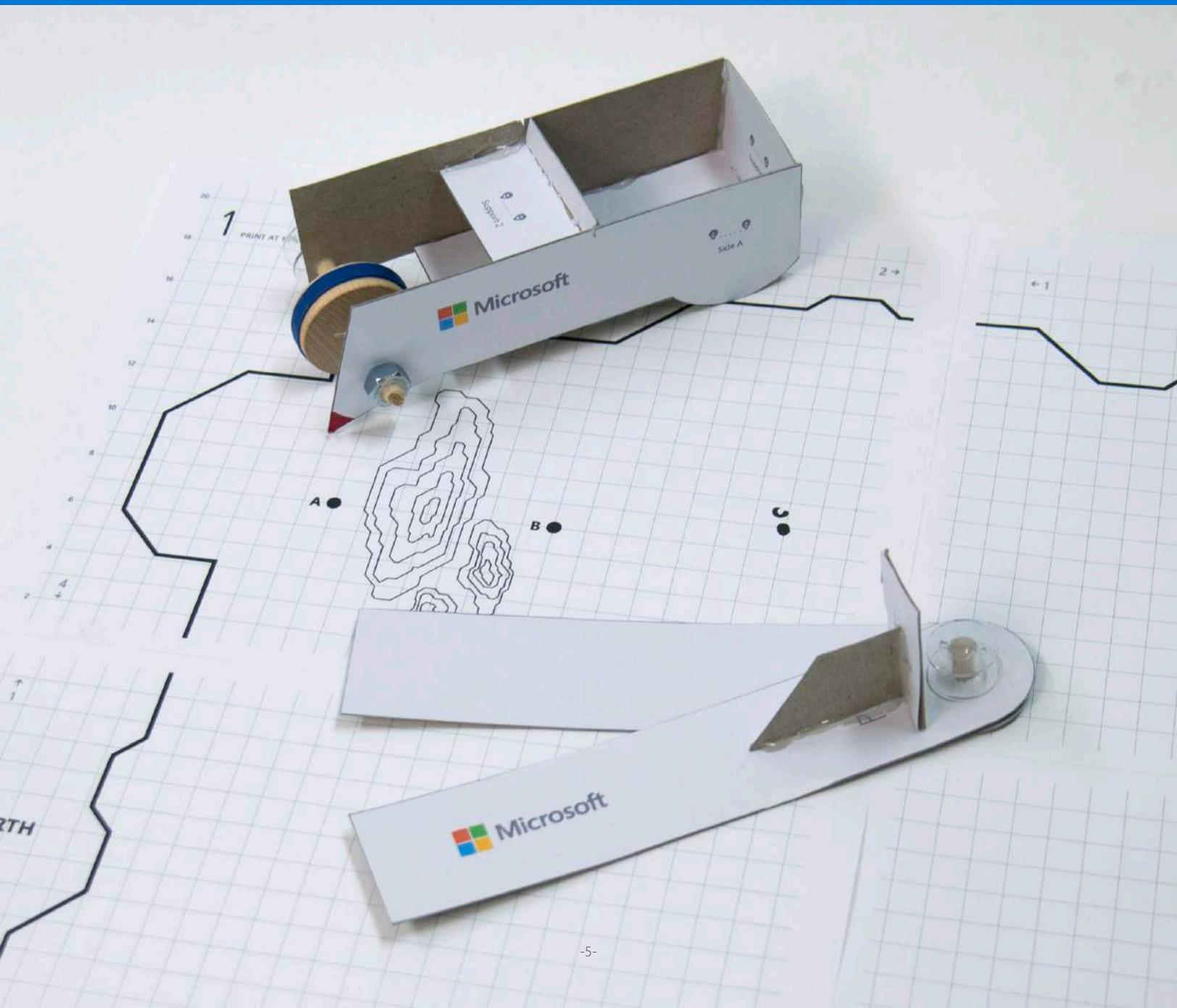
Click on the squares to visualize how the area for  $a^2$ , together with the area for  $b^2$  create the area for  $c^2$ .

Using the sliders, you can change the length of  $a$  and/or the length of  $b$  which, then, changes the length of  $c$ . This is shown in graphics and in numbers.

## PART 1

# Plan and Build

Students are placed into the roles of environmental surveyors and engineers. In this activity we make a map plan for a national park using the Pythagorean theorem and build surveying tools to help us digitize our map.





# Things You'll Need

 Get links to all the materials you need at: [aka.ms/measuringtoolsshoppinglist](https://aka.ms/measuringtoolsshoppinglist)

## Materials

- printed map template (pg. 32)
- print template (pg. 36)
- 3 sewing bobbins
- 130cm 1/4" dowel
- 1 wood wheel
- 1 3" rubber band
- 1 5/16" nut
- electrical tape
- 360 degree rotary encoder

## Toolkit

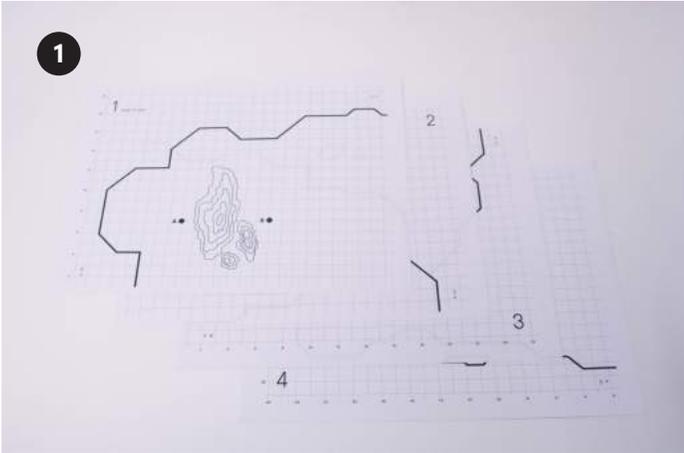
- scissors
- pen
- metal ruler
- glue stick
- hot glue gun
- hole punch
- wire strippers
- clear tape

Start with the technical requirements at:  
[aka.ms/hackingSTEM](https://aka.ms/hackingSTEM)

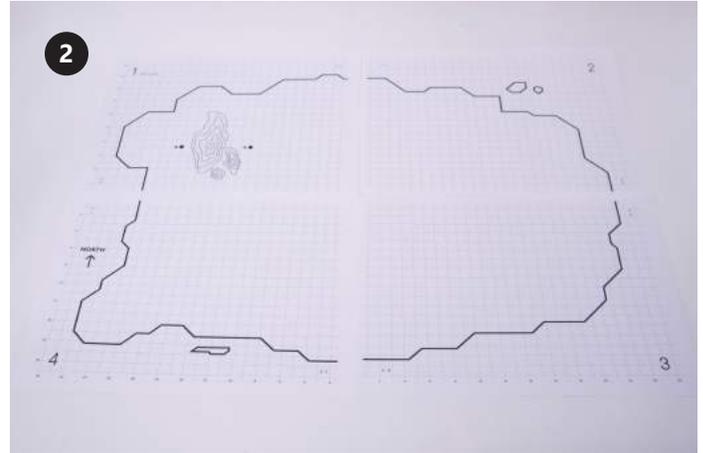


 This symbol indicates steps that require extra care and precision.

# Construct the map



Print out your map templates (pg. 32) at full size.



Organize your map so that all the edges line up with the right numbers.



Tape the edges of the map together using clear tape. Make sure the pages line up end to end with no overlapping.



Find point A and B and plot the remaining 6-8 points on your map following the constraints your teacher has provided.

# Prepare Parts for the Tools



1 Print out your template (pg. 35) at full size and obtain a flattened cereal box or thin cardboard piece.



2 With your glue stick, cover the entire back of the printed template with glue.



3 Lay the backside of the template over the cardboard and rub so it is flat.



4 Cut out all solidly lined pieces in the template. Do not cut the dotted lines.



5 Using your hole punch, cut out all areas where you see an outlined circle.



★ **Awesome!** Your parts are ready! Next you will build your measuring tools.

# Make an Angle Finder



Collect all the Angle Finder parts. They are marked with the Angle Finder symbol: 



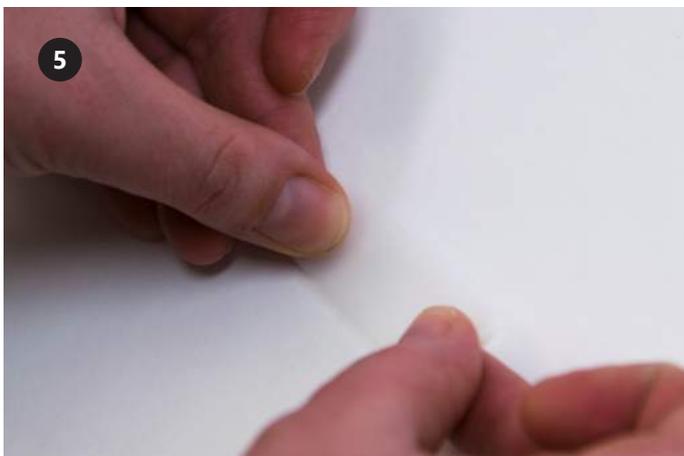
Next, mark your 1/4" dowel at 1.2 cm.



Score the dowel at the mark using the cutting blade of the wire strippers until it is easy enough to break with your hands.



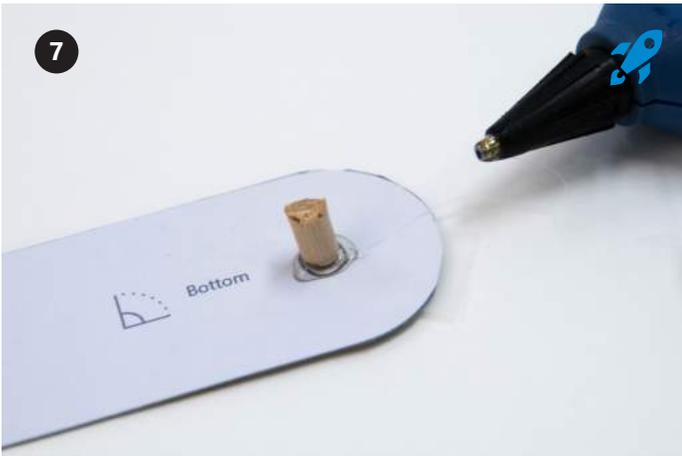
Breaking the dowel with your hands keeps the dowel from being crushed by the wire strippers and from flying off when cut.



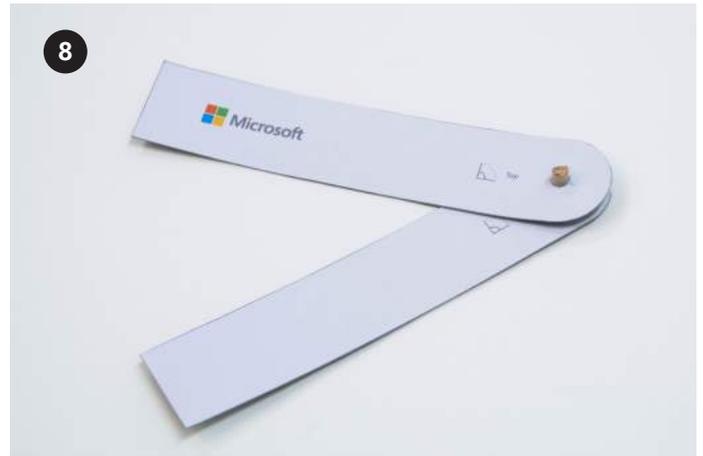
Place approximately 5 cm of clear tape on a flat surface.



Warm up your glue gun and then place the hole of the Angle Finder piece labeled "bottom" with the hole directly over the clear tape. The tape will act as a barrier between the glue and the table.



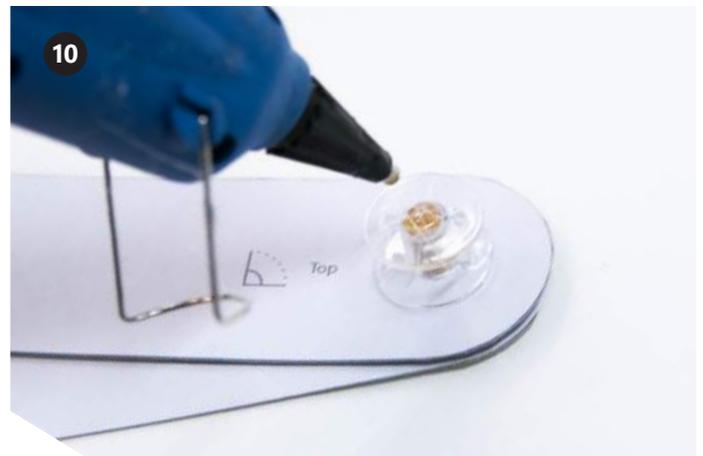
Next fill the hole with hot glue, and make sure there is glue on the sides as well. Then **quickly** push the dowel in and allow glue to **completely dry** before removing the Angle Finder from the tape.



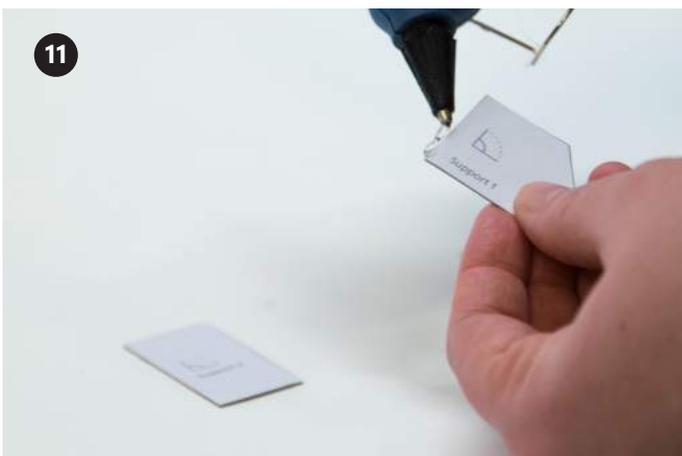
Place the top part of your Angle Finder over the dowel. This part will need to rotate freely.



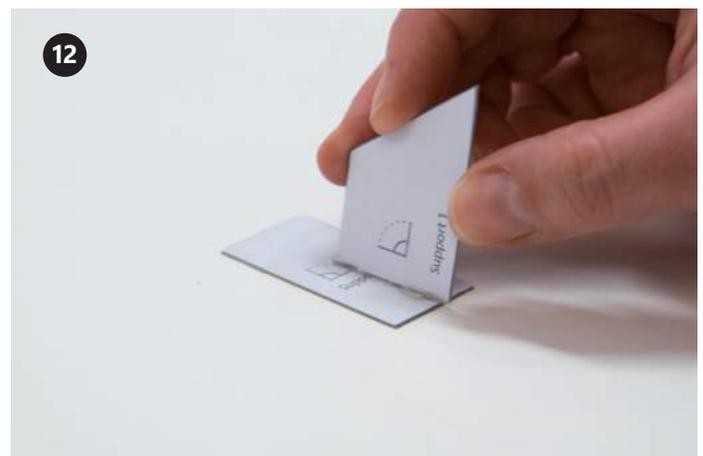
Place a new bobbin over the dowel so that it is flush with the Angle Finder template piece.



Secure the bobbin and dowel with a dot of hot glue through the center of the bobbin. Be sure no glue sticks out of the top of the bobbin.



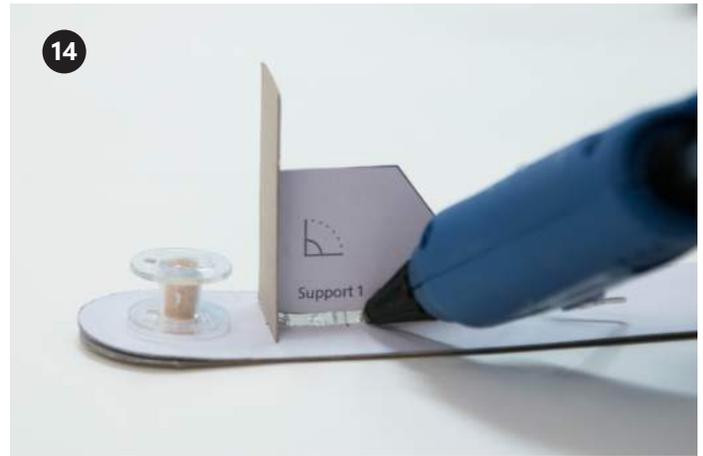
Now it's time to create a support for your rotary encoder. With your hot glue gun, run a thin line of glue along the side opposite the angled side.



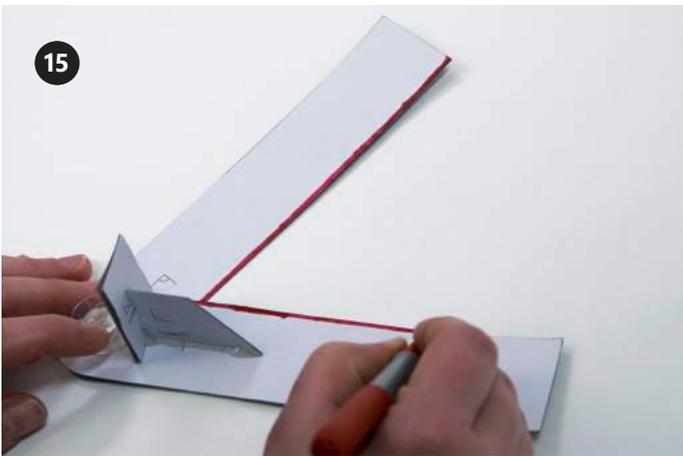
Glue Support 1 perpendicular to the center of Support 2 so the bottom of Support 1 and bottom of Support 2 are flush.



To find the most accurate place to glue your supports, use the encoder as a guide. Hold your encoder over the center of the bobbin and move Support 2 against the encoder.



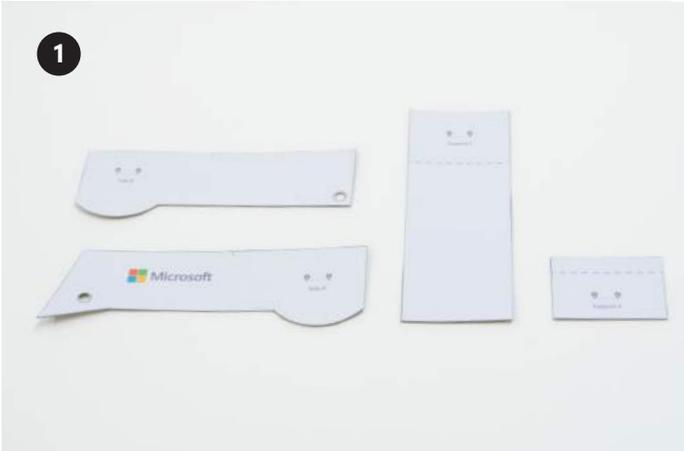
Move the encoder and glue down the base of the support.



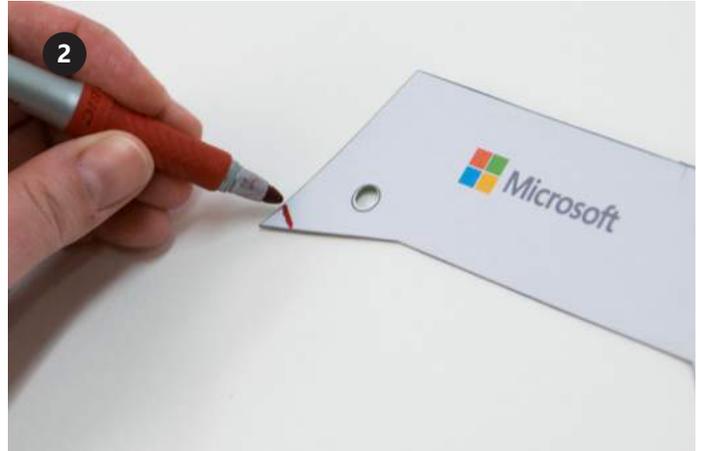
Hold your Angle Measurer with the bobbin closest to you, and rotate the top clockwise. Mark the interior edges of the angle tool as shown in the photo above. The marked areas should be on the left side of the top piece and the right side of the bottom piece.

★ **Awesome!** You've completed the Angle Finder. This component will also be used later in the activity!

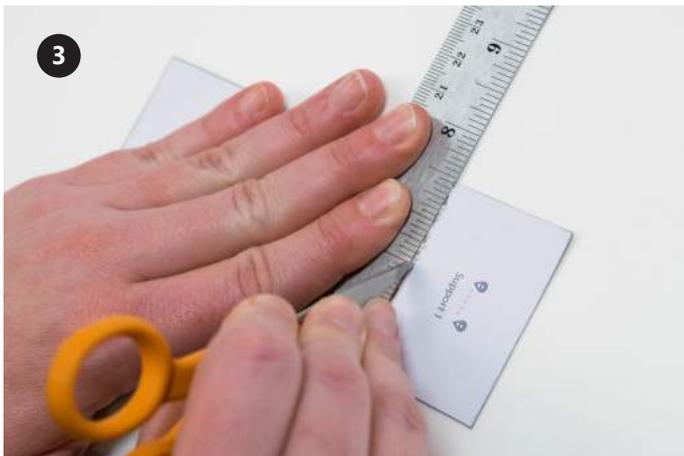
# Make a Distance Wheel



Gather all the Distance Wheel parts. They are marked with the Distance Wheel symbol: 📍.....📍



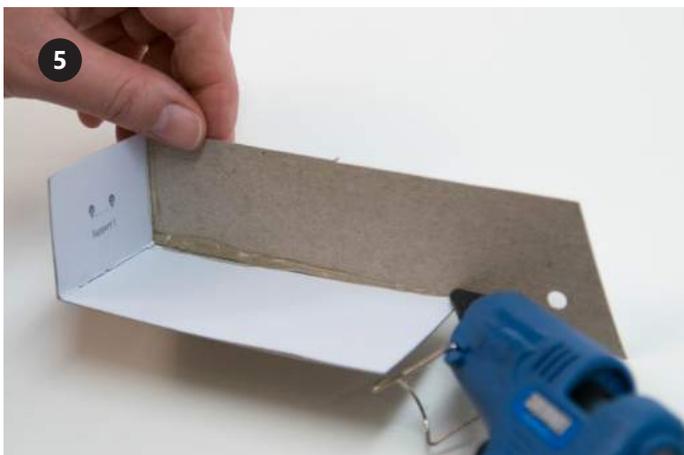
Take Side A of your Distance Wheel and lay it flat. Then mark the tip with a pen. This will be your guide for measuring.



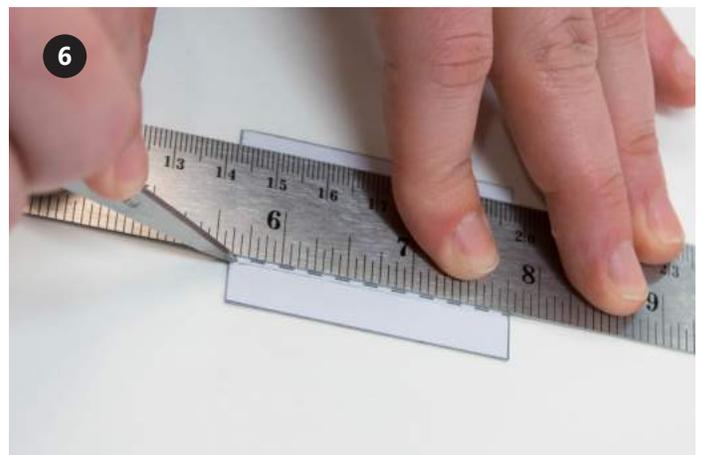
Take Support 1 and score it on the dotted line using a ruler and your scissors.



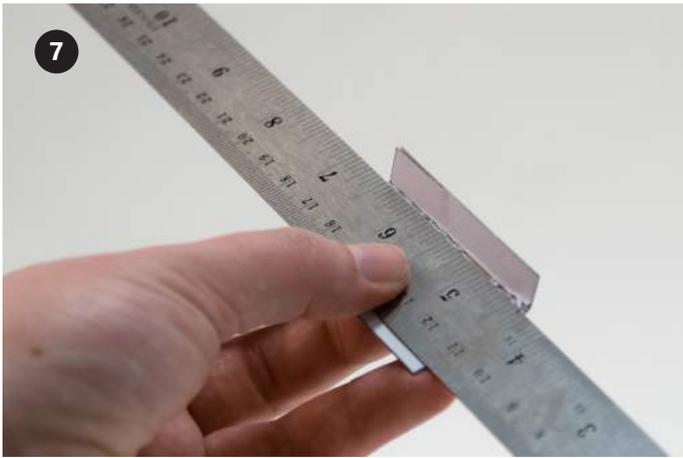
Bend Support 1 over your ruler at the scored line to give it a clean fold.



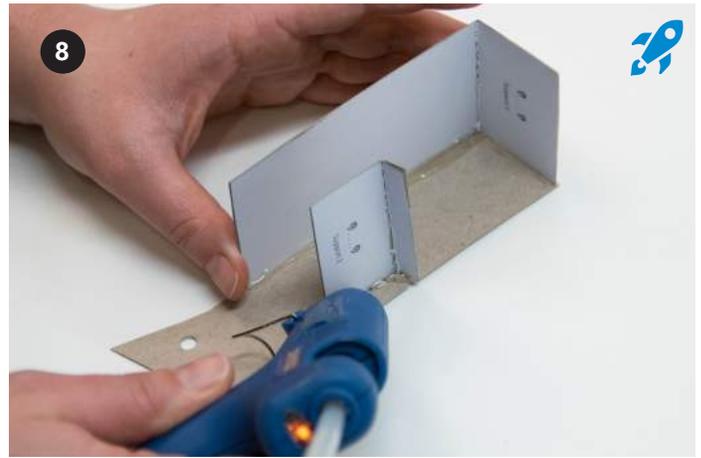
Line up Support 1 along the back edge of Side A. Hot glue Support 1 to Side A of the Distance Wheel as shown in the photo.



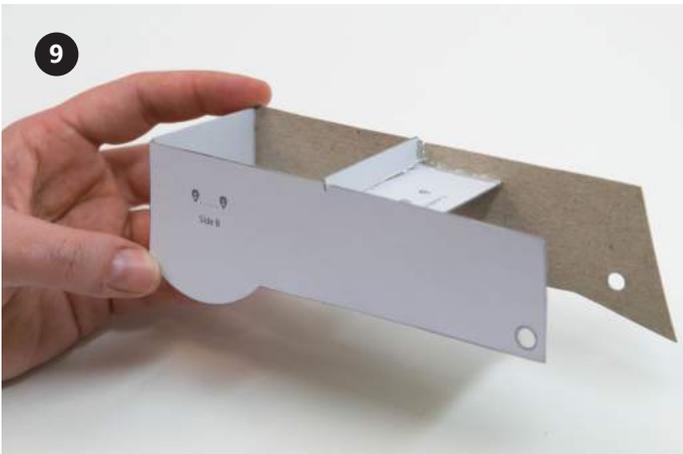
Score Support 2 the same as you did in step 3.



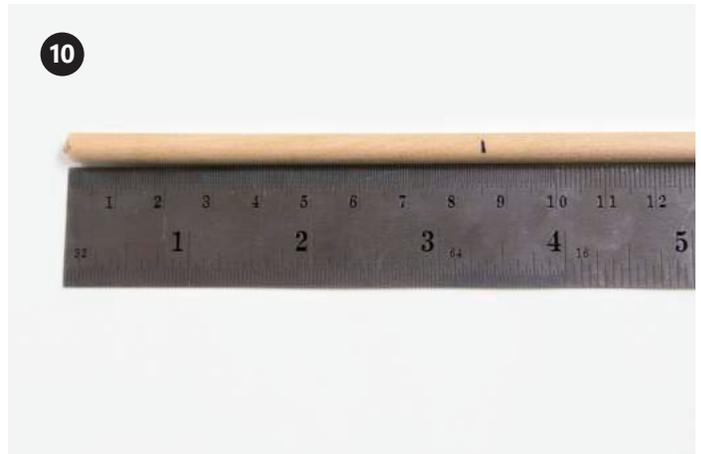
7  
Fold Support 2 at the scored dotted mark. It may be easiest to bend it around the ruler.



8  
Glue Support 2 at the notch mark in the middle of Side A.



9  
Once the glue has cooled on Side A, glue Side B to Support 1 and Support 2.



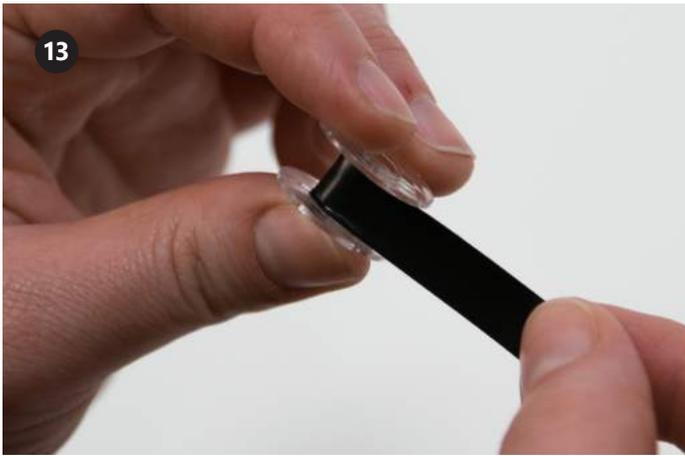
10  
Next, take your dowel and make a mark at approximately 8.5cm.



11  
Score the dowel at the mark using the cutting blade of the wire strippers until it is easy enough to break off with your hands.



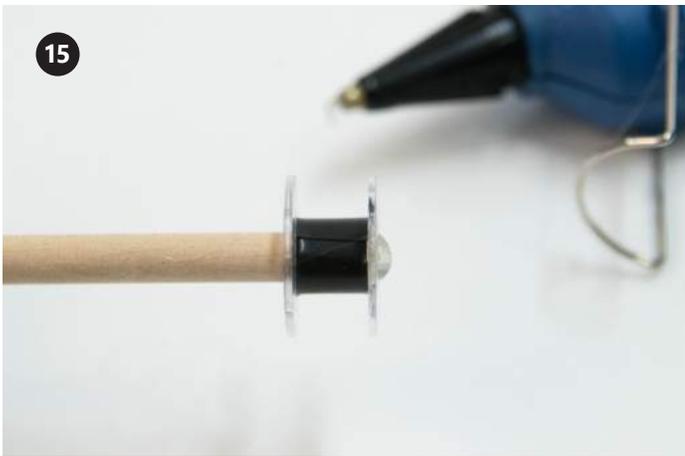
12  
Next, cut a 5 cm strip of electrical tape.



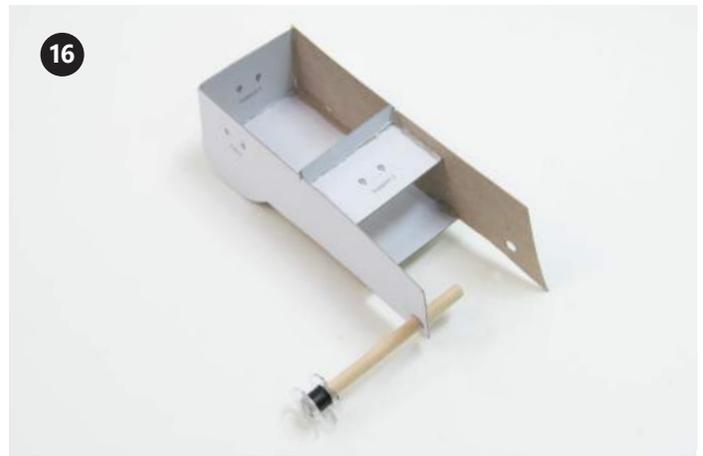
Wrap the electrical tape tightly around your bobbin.



Put a dot of hot glue in the center hole of the bobbin.



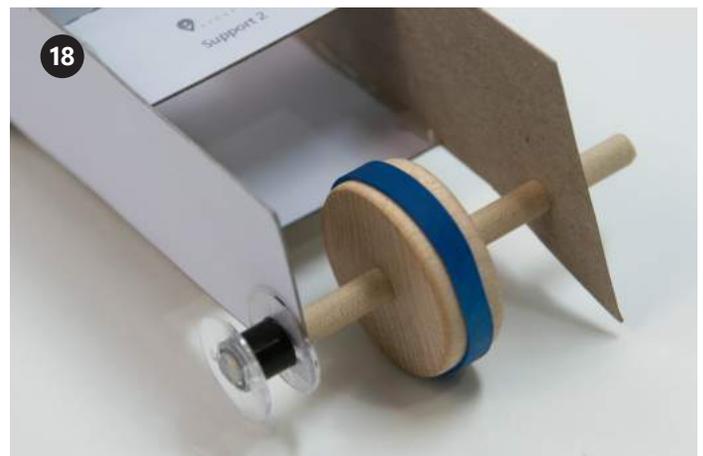
Push the dowel in through the bobbin and let dry. It is OK if the dowel sticks out a bit from the end of the bobbin.



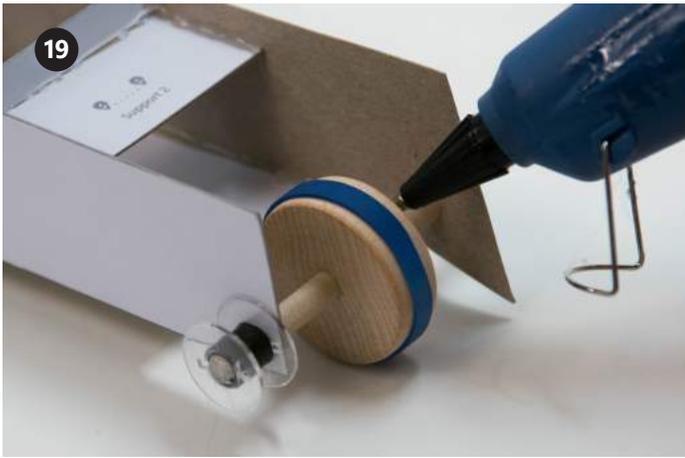
Put the dowel through Side B of your Distance Wheel, but not through Side A.



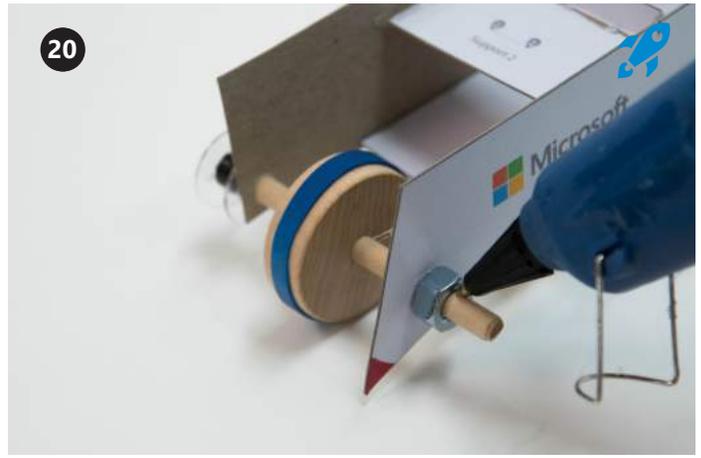
Find your wooden wheel and place the smaller rubber band around the wheel to give it grip.



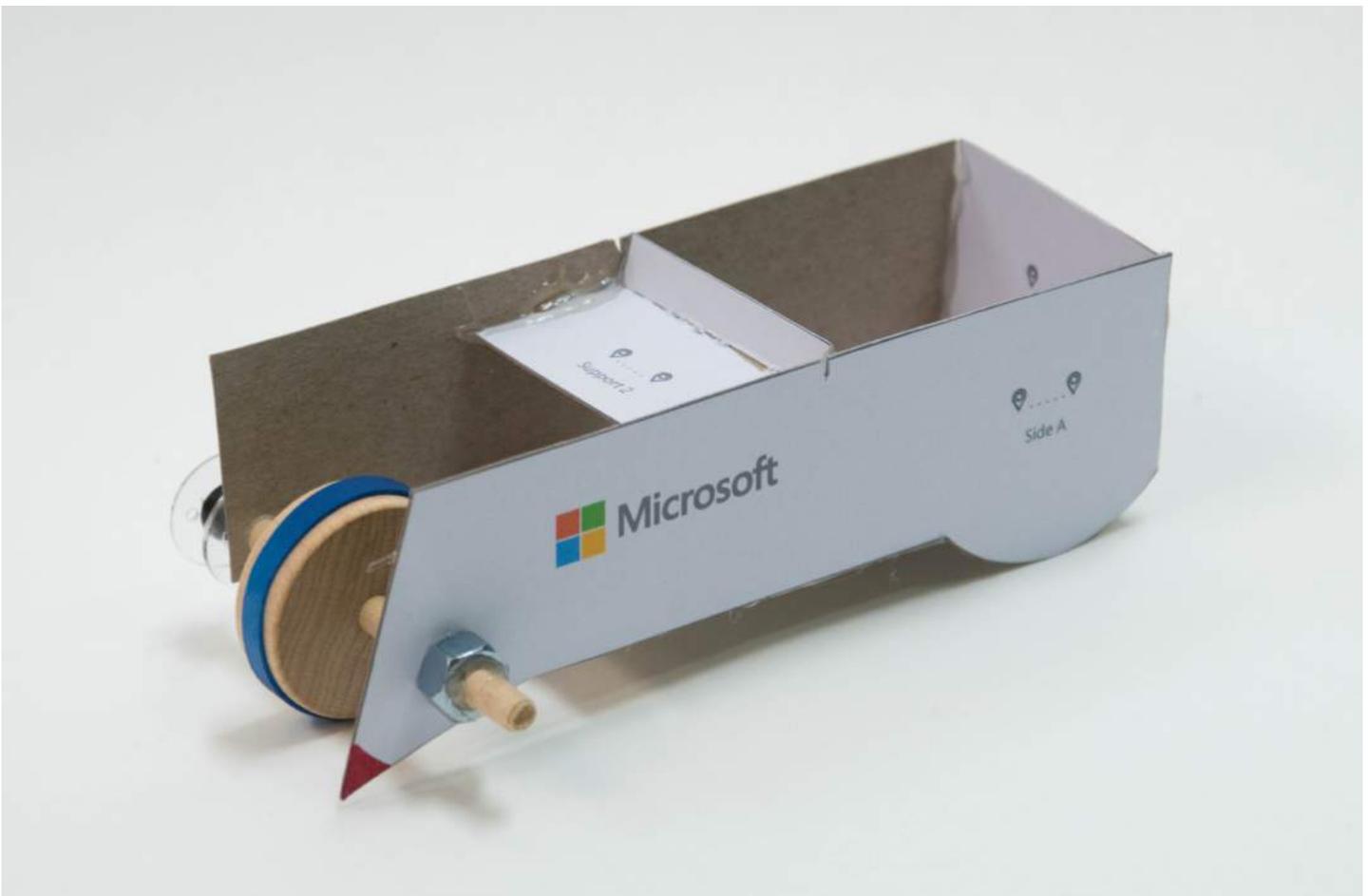
Put the wooden wheel on the dowel and put the dowel through Side A. Center the wheel between Sides A and B.



Secure the wheel to the dowel with hot glue on both sides.



Place the nut over the dowel on the side opposite the bobbin. Put glue on the exterior side of the nut so that no glue gets on Side A making sure the dowel and wheel spin freely.



★ **Great Job!** You've completed the Distance Wheel. You are now ready to connect your tools!

## PART 2

# Connect Your Device

Now that we have built our tools and mapped out our points, it is time to connect our tools in order to visualize the map. In this activity we will learn how to connect a rotary encoder to an Arduino microcontroller and upload code to that microcontroller.





# Things You'll Need

 Get links to all the materials you need at: [aka.ms/measuringtoolsshoppinglist](https://aka.ms/measuringtoolsshoppinglist)

## Materials

- 1 rotary encoder
- 1 Arduino Uno microcontroller
- 1 USB cable type A to type B
- masking tape
- electrical tape

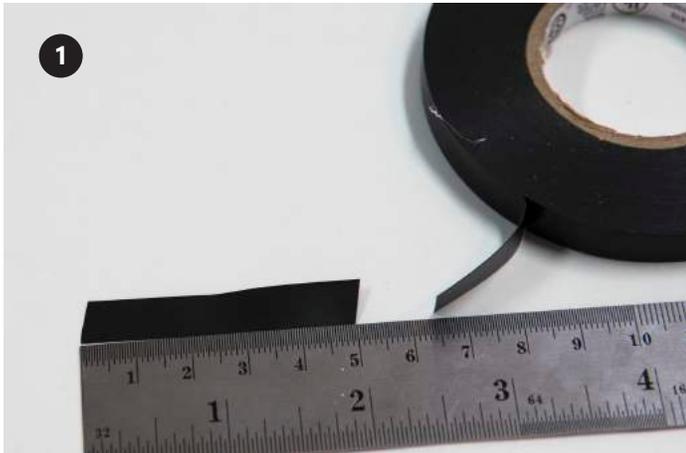
## Toolkit

- scissors
- ruler
- wire strippers
- hot glue gun

Start with the technical requirements at:  
[aka.ms/hackingSTEMmeasuringtools](https://aka.ms/hackingSTEMmeasuringtools)



# Prepare Your Encoder



1 Cut a 5 cm piece of electrical tape.



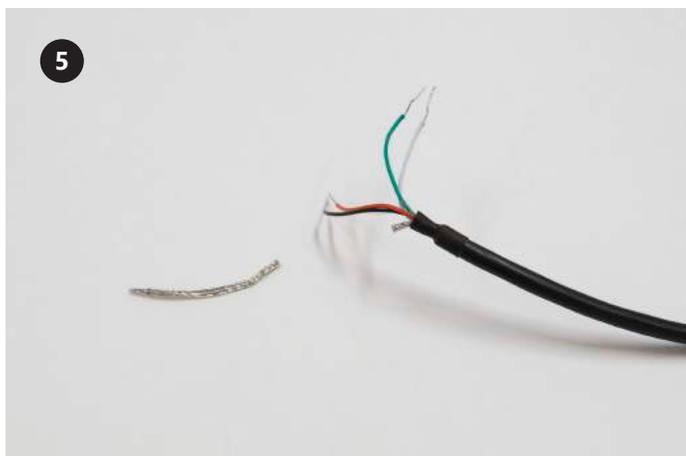
2 Wrap the tape carefully around your bobbin.



3 Put a small dot of hot glue on the flat part of encoder rod.



4 Push the bobbin onto the encoder's rod. The rod should be halfway through the center of the bobbin. Make sure no glue comes out of the top.

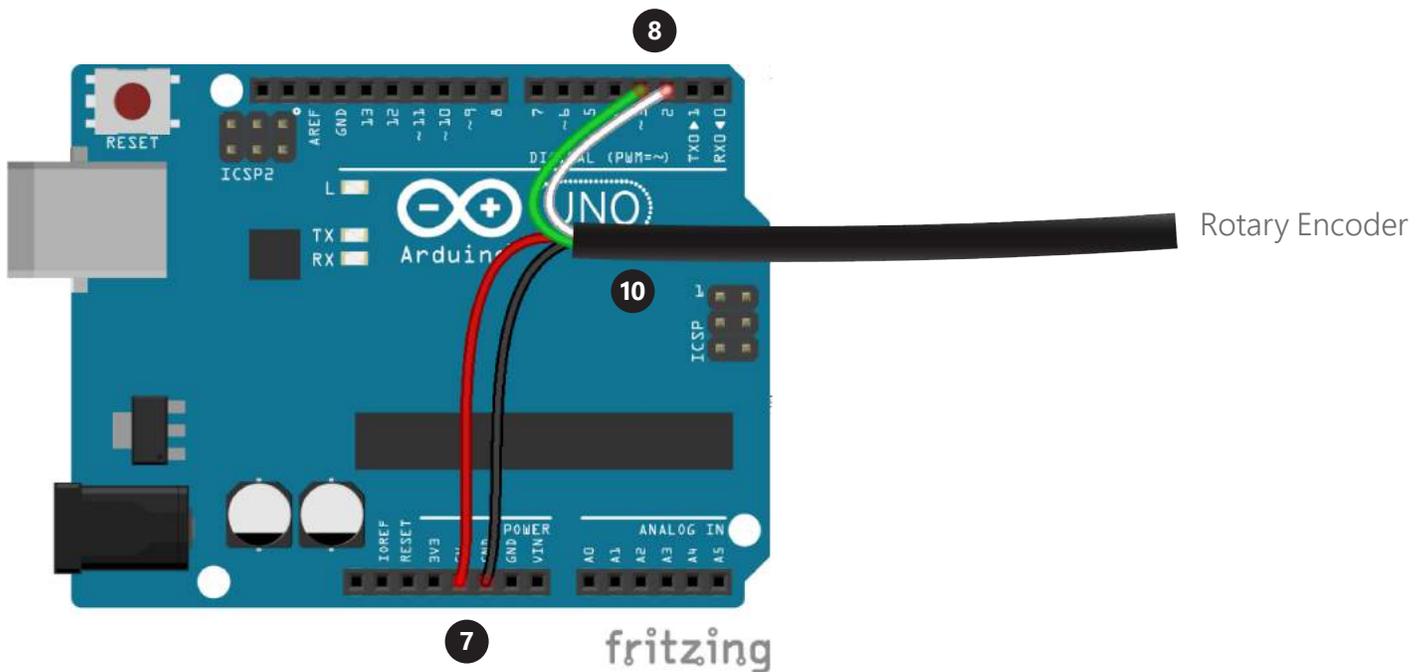


5 Cut off the silver braided metal wire with your wire cutters.



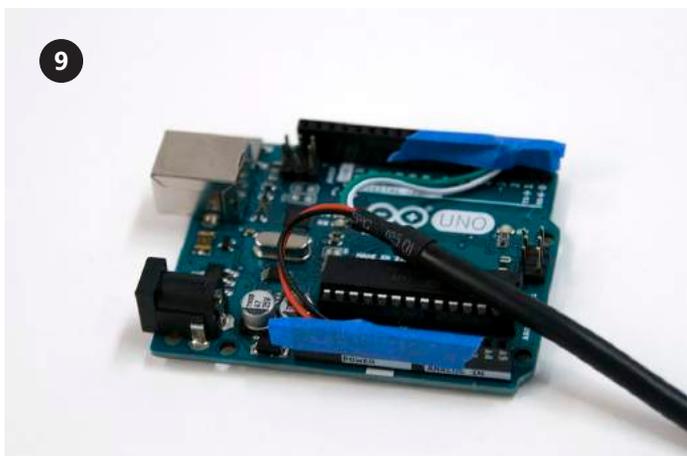
★ **Good Work!** You've completed the encoder. You now ready to connect it to your Arduino!

# Connect the Arduino



**7** Connect the rotary encoder by placing the black wire into the GND pin hole and the red wire into the 5V pin hole on the Arduino.

**8** Now insert your input wires into the Arduino by inserting the white wire into the 2 pin hole and the green wire into the ~3 pin input.



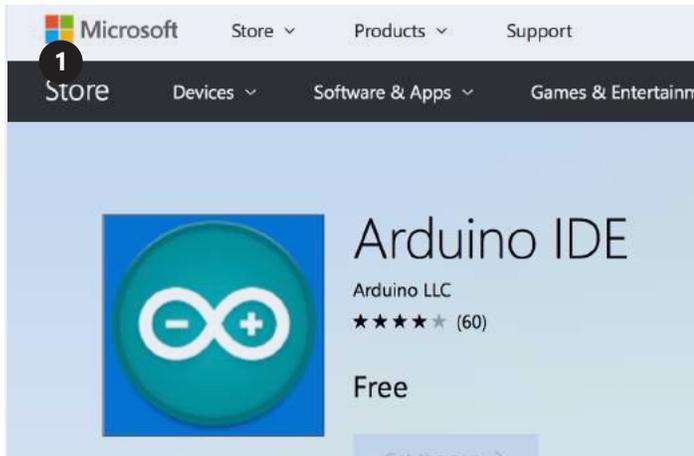
Your wires will loosen from the pin holes so we recommend applying a thin strip of masking tape over the pin holes to keep the wires from jarring loose.



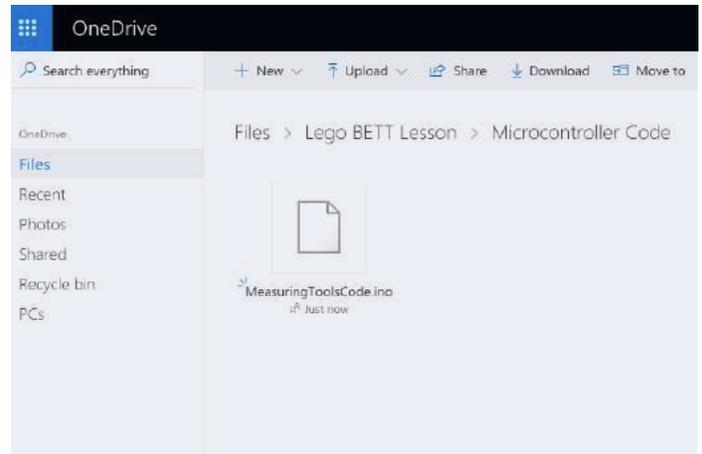
**Optional:** For extra support, we recommend applying a bridge of hot glue on the larger black wire attaching it to the microcontroller. This will not harm the microcontroller.

★ **CONGRATULATIONS!** Move on to uploading the code to your Arduino to begin visualizing data in Excel.

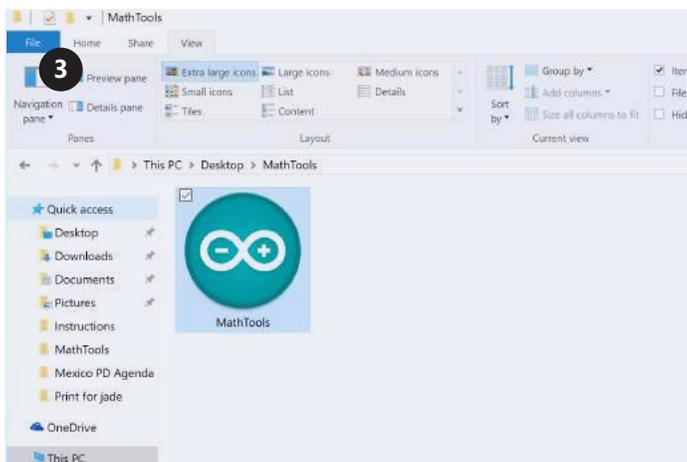
# Get the Flash Code



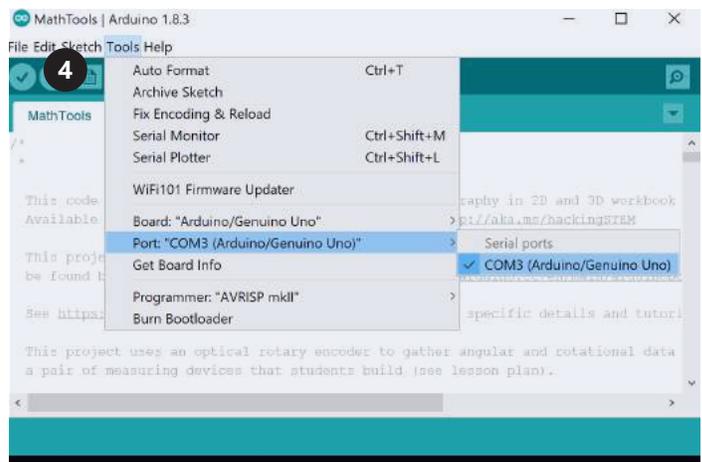
Start by connecting your Arduino to your computer with the USB cable. Next, you will need to install the Arduino IDE, which you can access through the Technical Requirement links at: [aka.ms/hackingSTEMMeasuringTools](https://aka.ms/hackingSTEMMeasuringTools)



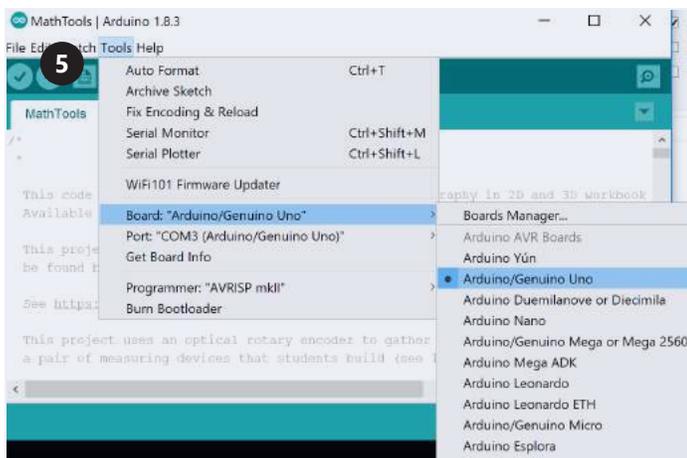
Go to [aka.ms/measuringtoolsflashcode](https://aka.ms/measuringtoolsflashcode) and download the flash code.



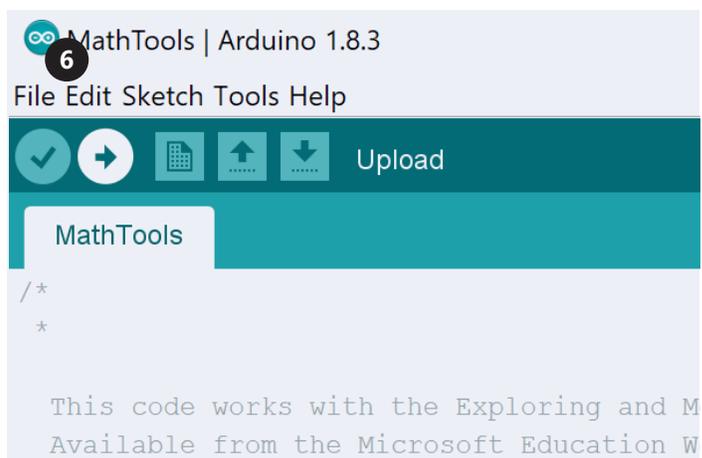
Open your downloaded file to launch the Arduino app.



Next, select: Tools > Port > COM3 (Arduino Uno) Your com port may be different than shown.



Then select Tools > Board: "Arduino/Genuino Uno" > Arduino/Genuino Uno.



Click on the circular right arrow button to upload.

# Get Project Córdoba

## To run project Córdoba, make sure you meet these technical requirements:

- PC running Windows 10, and Excel 2016 (Desktop)
- Project Cordoba Add-In: Update your existing copy of Microsoft Excel 2016 with a free add-in to support real-time data streaming from your projects available at: [aka.ms/getaccess](https://aka.ms/getaccess)

Once you have downloaded Cordoba, open up Excel and get yourself acquainted with the UI:

To connect your Arduino, you need to plug in your device to your computer via USB and then click "Connect a Device"



File	Home	Insert	Page Layout	Formulas	Data	Review	View	Project Córdoba					
1	Connect A Device	OR	Import Data File	2	Start Data	Stop Data	3	Record Data	Stop Recording	Capture Visualization	Reset Data	Advanced	Help
	Data Sources			Data Streaming			Data Recording		Advanced		Help		

Once your device is connected, select "Start Data" in order for data to begin streaming into Excel. If you do not click "Start Data" when your device is plugged in, you will not see any live data.

If you have recorded and saved a data file (.csv), you can import it with this button.

## PART 3

# Visualizing the Data

We are now ready to transfer our analog, physical map into digital 2D and 3D models. In this activity we will attach our rotary encoder to our tools and plot our points into a custom Excel workbook which will then export the map to a file that can be placed in Paint3D so we can begin to discuss z-space.

Angle

BCD = 2°

0°

330° 30°

300° 60°

270° 90°

240° 120°

210° 150°

180°

RESET SAVE

ABC 232° FGH 153°

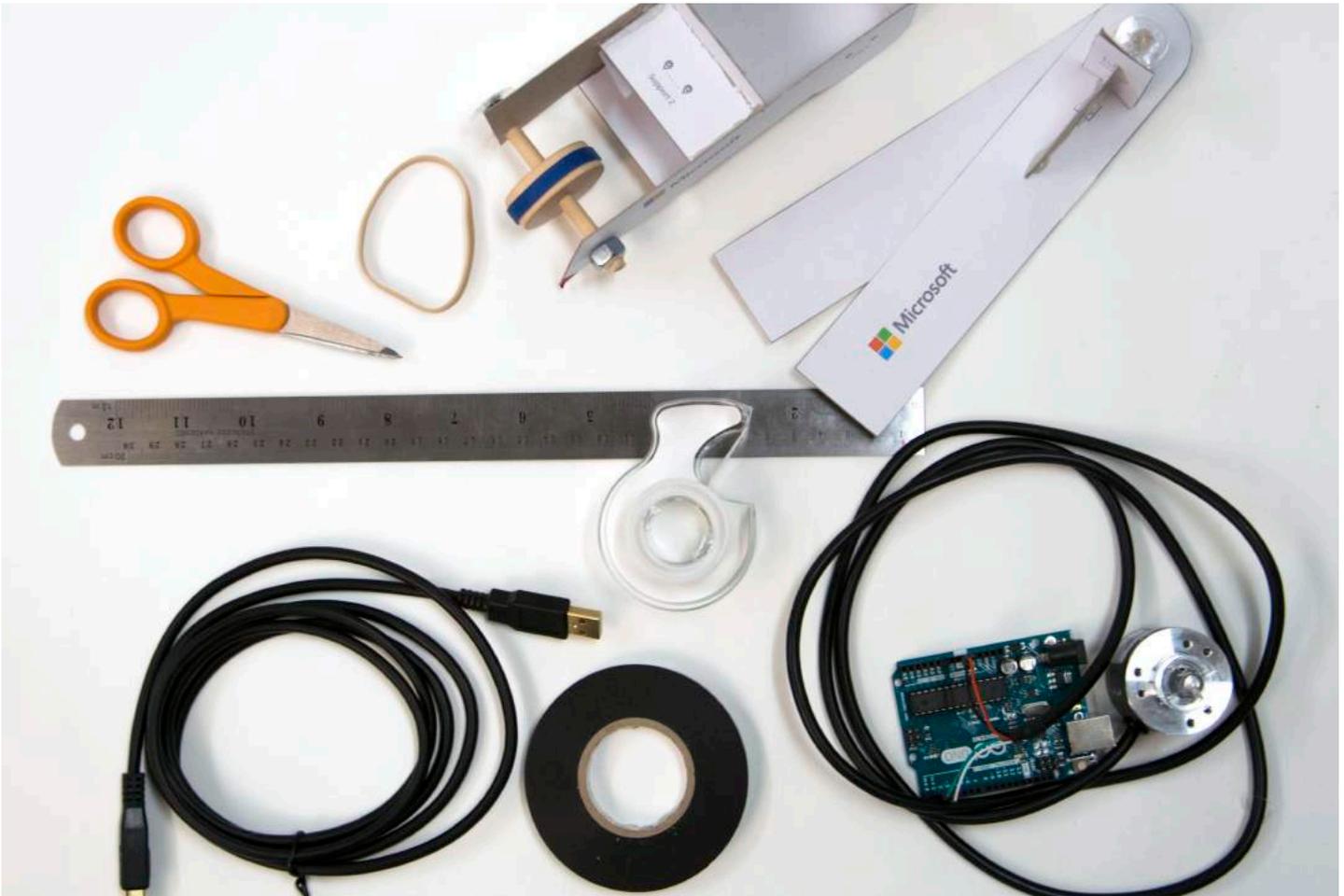
BCD 128° GHI 61°

CDE 153° HIJ 293°

DEF 230° IJA 50°

EFG 78° JAB 62°

OUT OF ROAD



# Things You'll Need

 [Get links to all the materials you need at: aka.ms/measuringtoolsshoppinglist](https://aka.ms/measuringtoolsshoppinglist)

### Materials

- completed cardboard tools
- 1 modified & connected rotary encoder
- 1 USB cable type A to type B
- electrical tape
- 1 3.5" rubber band

### Toolkit

- scissors
- ruler
- clear tape

**Start with the technical requirements at:**  
[aka.ms/hackingSTEMmeasuringtools](https://aka.ms/hackingSTEMmeasuringtools)



# Download the Workbook

To complete the full project, make sure you meet these technical requirements:

- PC running Windows 10, and Excel 2016 (Desktop)
- Project Cordoba Add-In: Update your existing copy of Microsoft Excel 2016 with a free add-in to support real-time data streaming from your projects available at: [aka.ms/getaccess](http://aka.ms/getaccess)
- Customized Excel Workbook available at: [aka.ms/measuringtoolsworkbook](http://aka.ms/measuringtoolsworkbook)

## Tool Setup Worksheet

Make sure to connect your device and click "Start Data" to begin viewing live data!

Click the radio buttons to begin the trials. Make sure you are using the correct tool when trying to calibrate the distance or practice your angles.

Click "Calibrate" once you have successfully completed all 4 trials. This will calculate the correct radius of your distance wheel.

Navigate your worksheets by clicking on the bottom tabs. The highlighted tabs will be used for the lesson.

Workbook Settings

Settings below are teacher inputs into the workbook that are needed

AB Height (cm)	10
max road length (cm)	150
Encoder Points Per Revolution	360
# Encoders Used	1

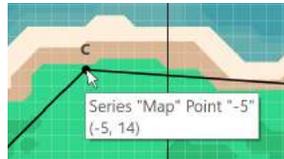
The "Teacher" tab is used for adjusting the settings of the physical map and the points per revolution (PPR) of the encoder.

The "SET TO 0°" button in the workbook will reset your tool back to 0°. It is a good practice to reset your tool after every measurement to record accurate data.

"Clear All" will remove all data and trials from this worksheet

# Dashboard Worksheet

Connect Your Device and click "Start Data" to begin viewing live data!



Hover over your points to see their coordinates. This is a good way to check if you have plotted your points accurately.

Set your number of points in the workbook to the number of points on your physical map.

Select the radio buttons on the worksheet to measure the angle or distance on your map.

Don't forget to check out the Big Data activity!

The "SET TO 0°" button in the workbook will reset your tool back to 0°. It is a good practice to reset your tool after every measurement to record accurate data.

The "Save" button in the workbook records your tools data into the corresponding tables.

The "Road Length" section shows the total length of your road in centimeters. You can use this area to see the ecological impact of building shorter or longer roads.

The "Clear Map" button erases the entire map of all your points, angles and distances.

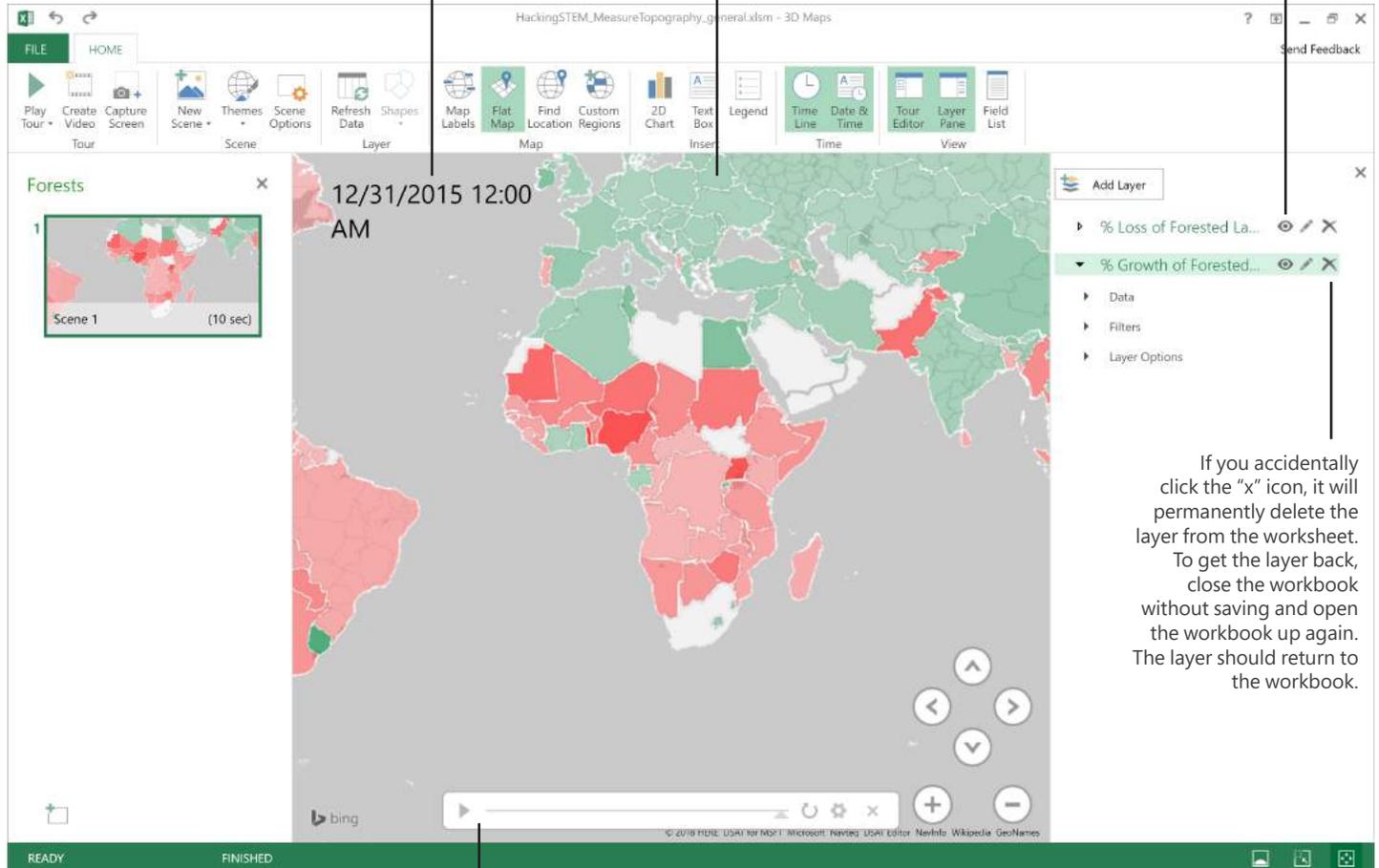
"Export map to 3D paint" exports your map as a png onto your desktop.

# Global Deforestation over Time

- Net Growth of Forested Areas
- No Net Growth or Loss of Forested Areas
- Net Loss of Forested Areas

Click the eyeball icon to toggle the layer on or off

The timestamp for the scene



If you accidentally click the "x" icon, it will permanently delete the layer from the worksheet. To get the layer back, close the workbook without saving and open the workbook up again. The layer should return to the workbook.

Press the grey triangle to play the timeline or to scrub through the timeline scene

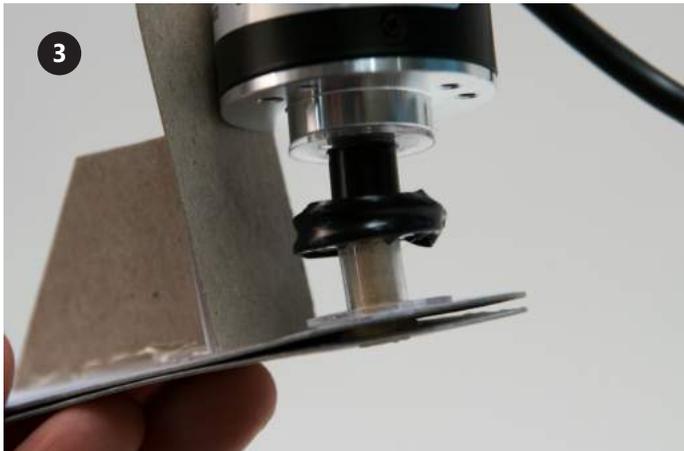
# Attach the Encoder to the Angle Finder



1 Cut a 10 cm piece of electrical tape.



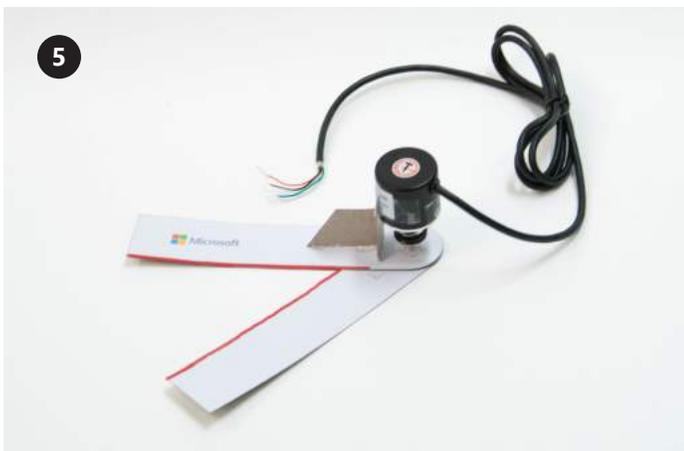
2 Line up the encoder and bobbin on top of the Angle Finder bobbin and tape them together using the 10 cm of electrical tape.



3 Make sure the tape is holding the two bobbins firmly together.

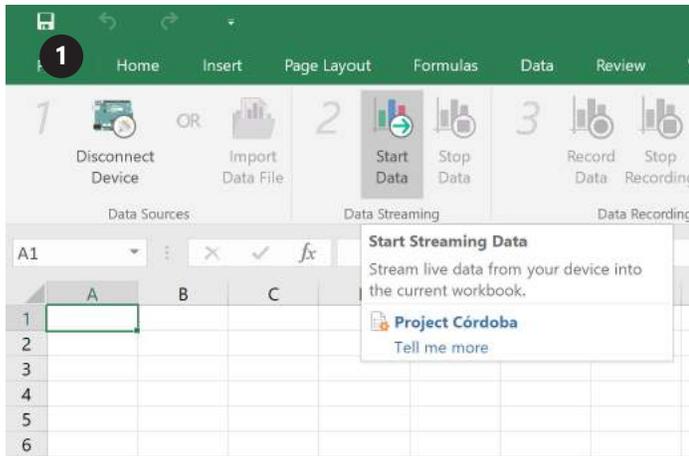


4 Tape the top of your encoder around the support of your Angle Finder using some clear adhesive tape.

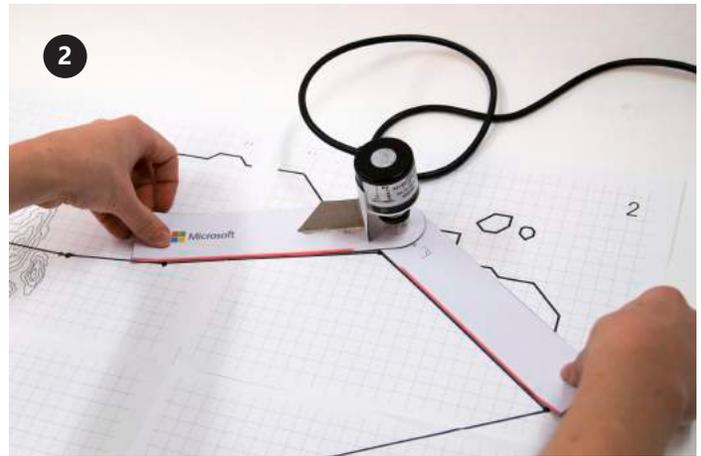


★ **Good Work!** You've attached the encoder to your Angle Finder. This component will also be used in the next activity!

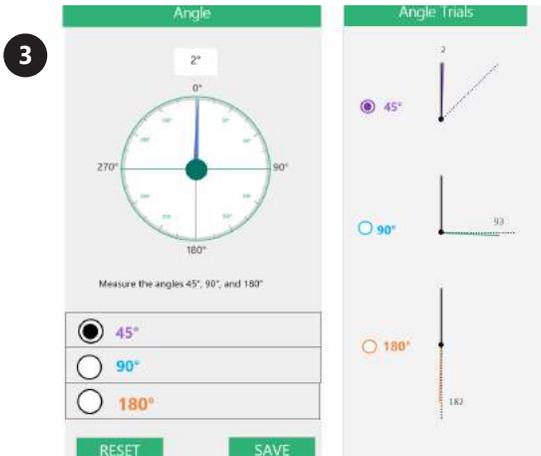
# How to Use the Angle Finder



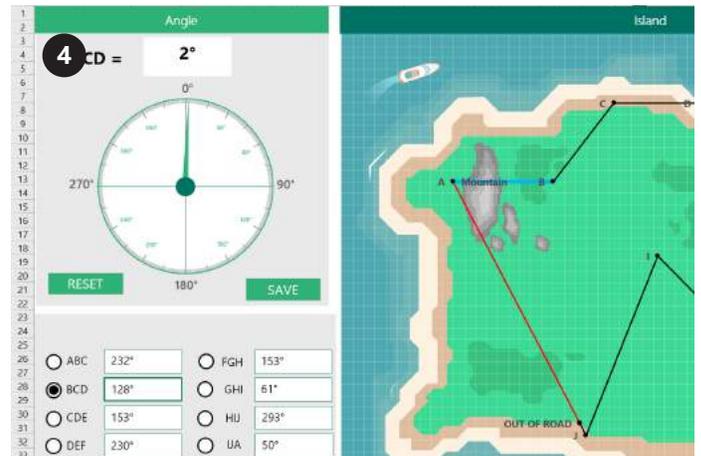
Once connected to the computer, make sure you connect your device on Project Cordoba and Select "Start Data".



To measure the angles correctly, line up the bottom of the angle tool with the line segment that the angle is leading to and the top of the angle tool will rotate to the line segment that you are coming from.



Practice measuring angles in the Tool Setup dashboard. Click "Save" once you're satisfied with the angle. It is also helpful to click "Reset" every few measurements to ensure accurate data.



Once you have the angle measuring down, move to the Island Dashboard sheet and begin entering in your angles on the Excel Map using the Angle section of the worksheet.

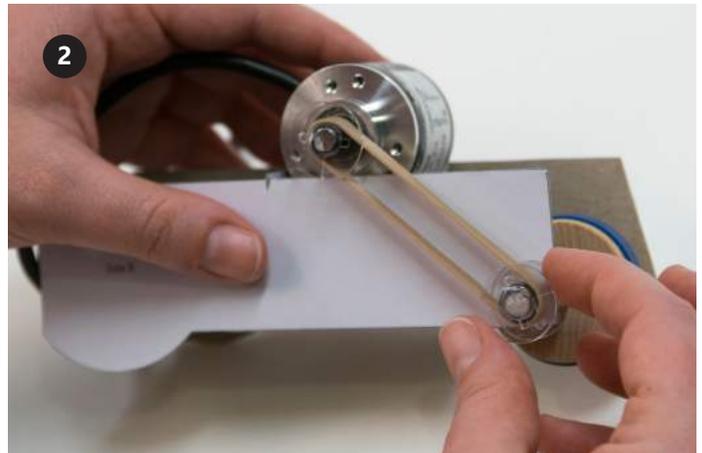


Once you are finished and satisfied with entering your angles into the worksheet, remove the rotary encoder from the Angle Finder so you can use it for the distance wheel.

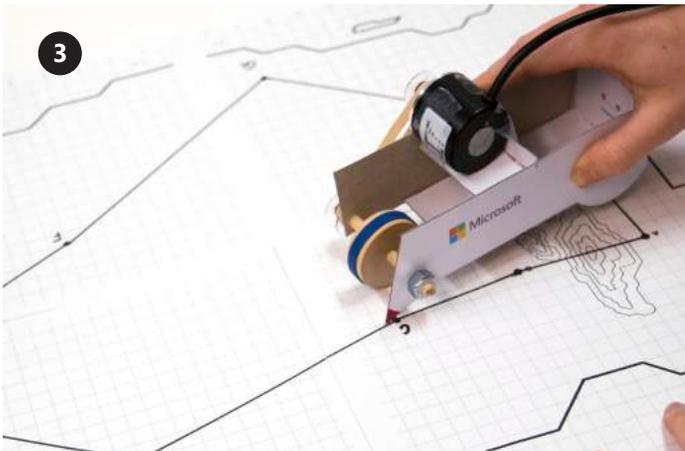
# Attach and Calibrate Distance Wheel



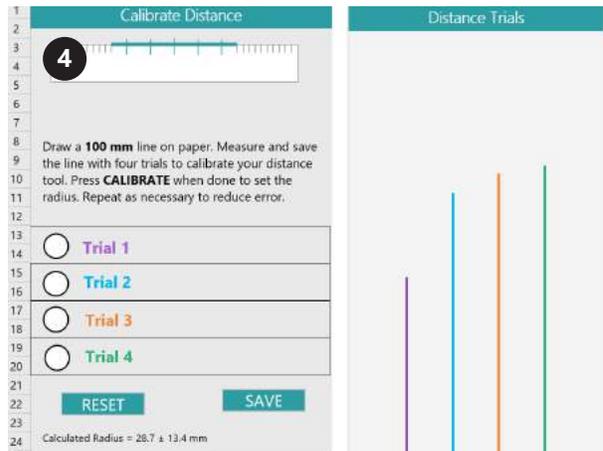
To attach the encoder to the Distance Wheel, first place the encoder on Support 2 with the bobbin on the exterior of the distance tool and secure it with clear tape.



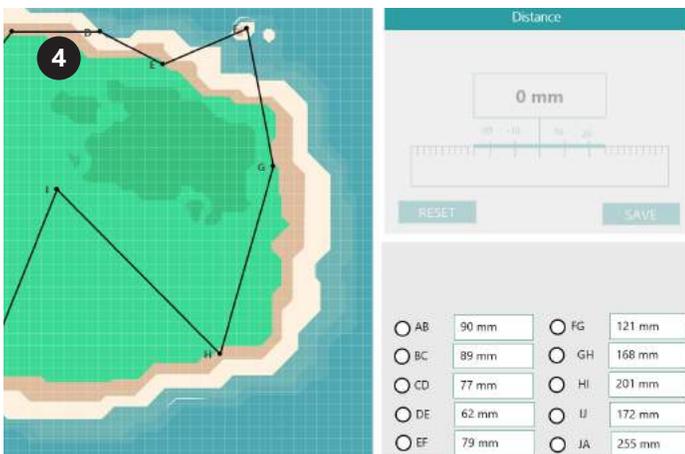
Then, put your 3.5" rubber band around the two bobbins.



To correctly measure with the Distance Wheel, line up the red marked pointer with the line you are measuring and continue the pointer straight on the line.



Calibrate your distance tool in the "Tool Setup" workbook before moving into the Island Workbook.

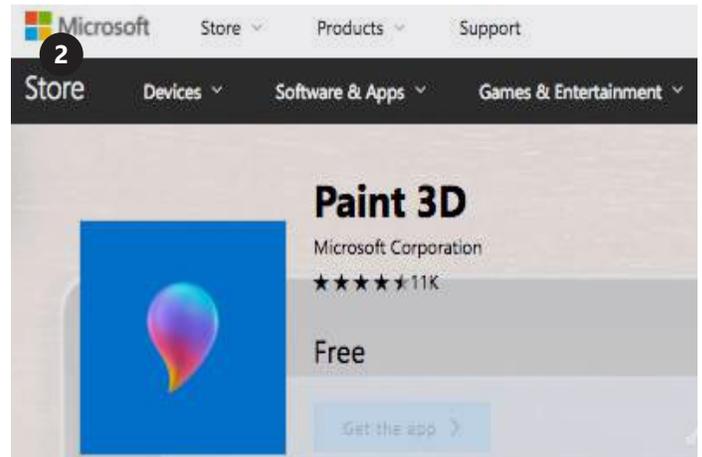


Once you have the Distance Wheel calibrated, move to the Island Dashboard sheet and begin entering in your distances on the Excel Map using the Distance section of the worksheet.

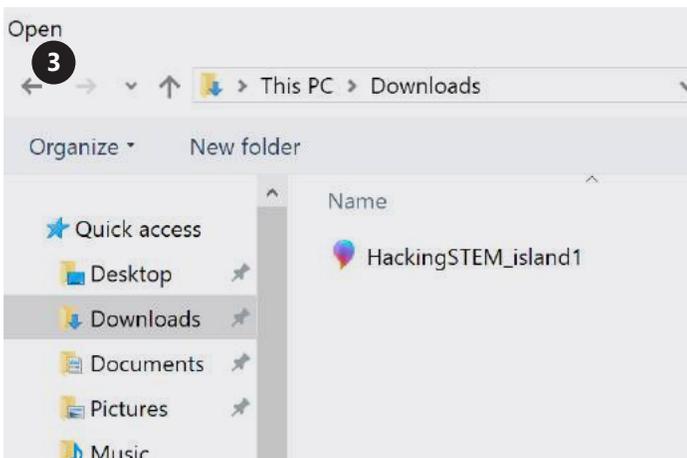
# Paint 3D Basics



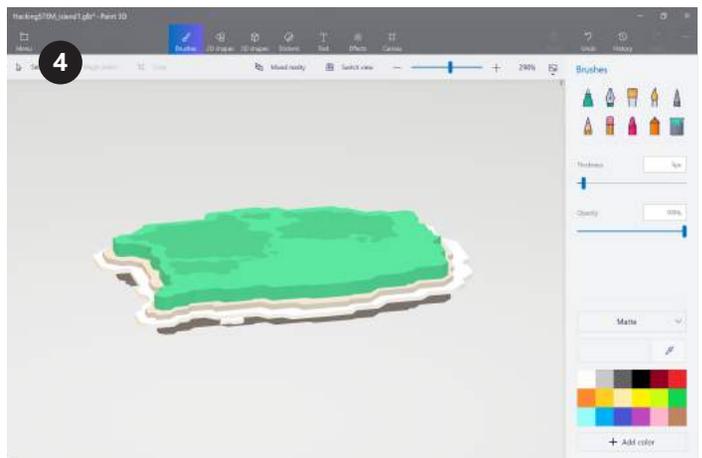
Export your map from the Excel worksheet.



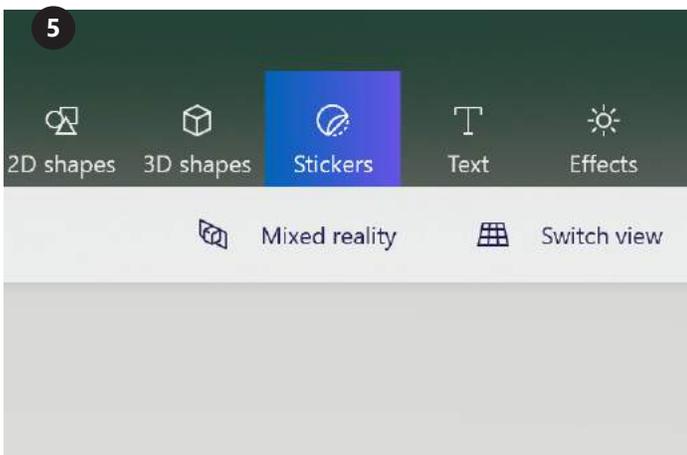
Download and open Paint 3D.



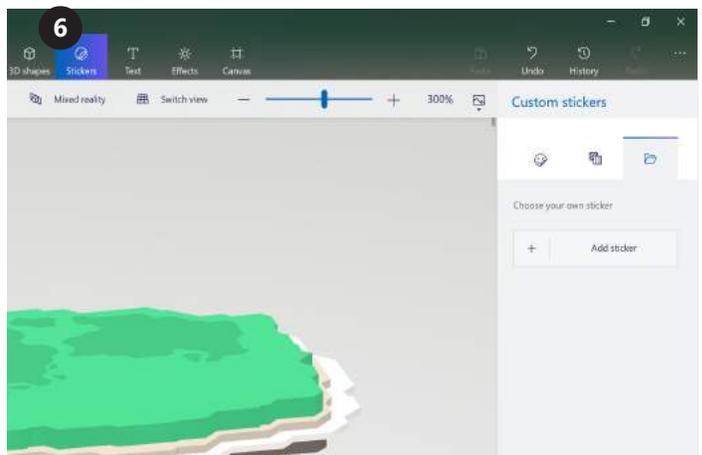
Open the file HackingSTEM\_island from the lesson download.



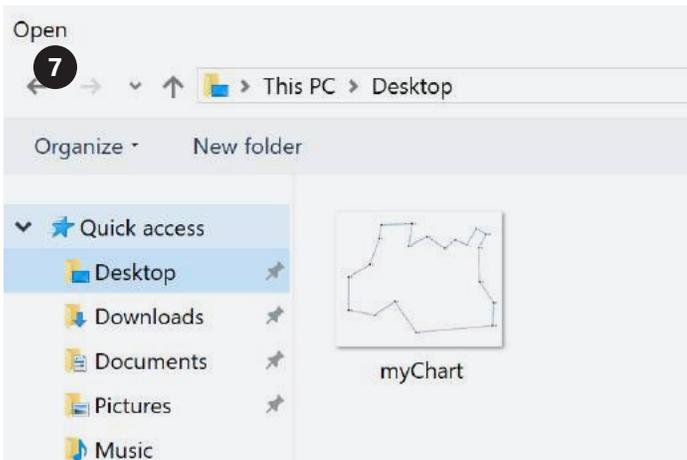
If this is your first time using Paint3D, we recommend using online tutorials and taking time to explore the app.



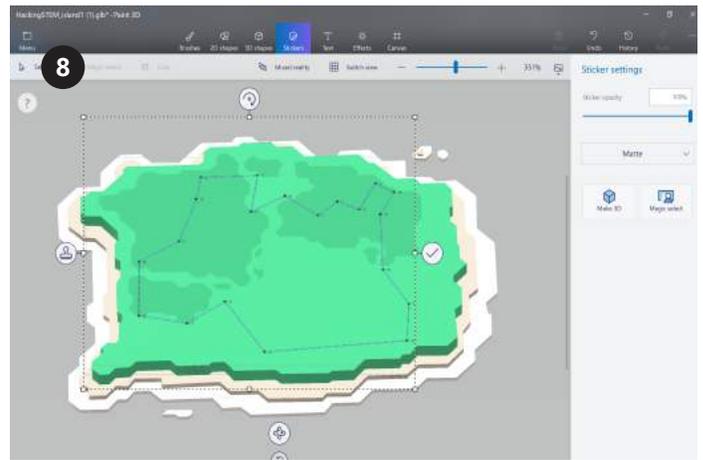
To get your map on the island, you will need to create a sticker which can be selected from the top menu.



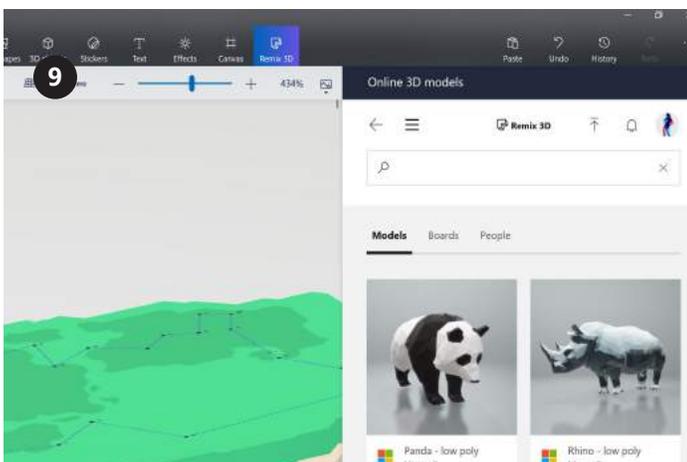
A menu will appear on the right side. Select the folder tab to add a sticker.



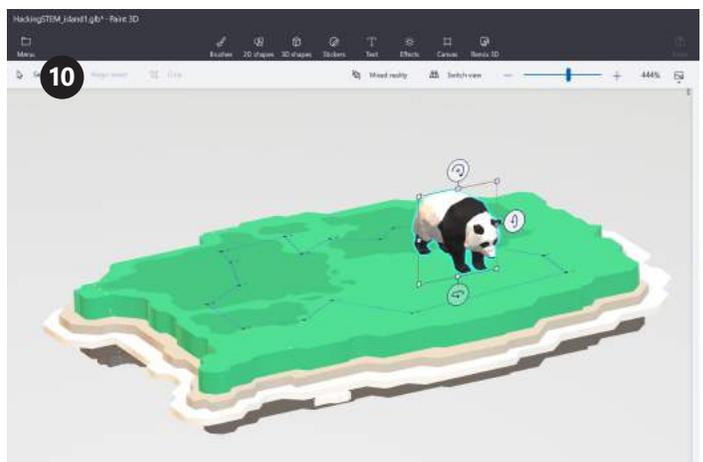
Find your saved map and select to insert.



Use the controllers to place your map onto the island. Use the undo button if it does not land where desired.



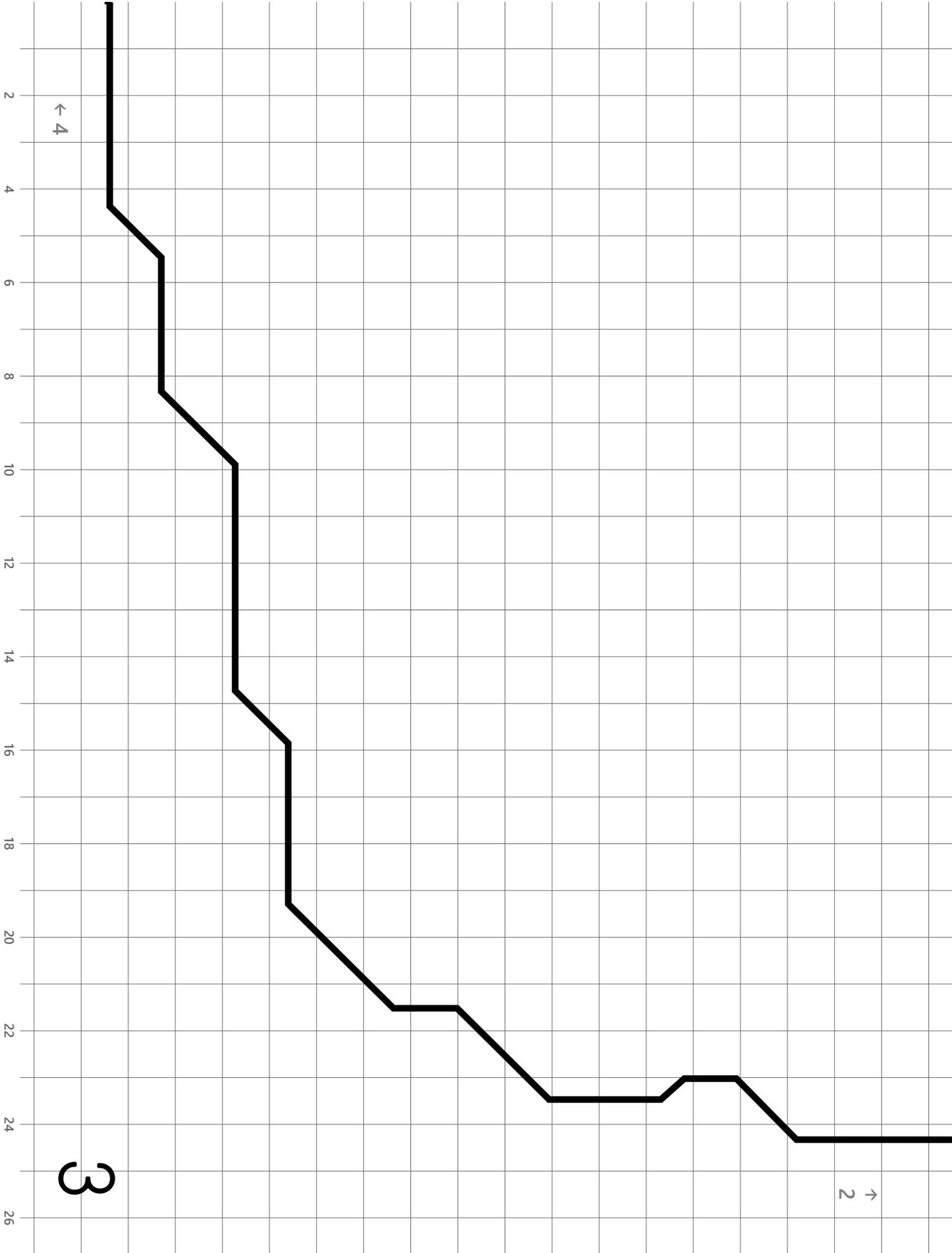
Select Remix 3D to insert elements to your map or create your own!



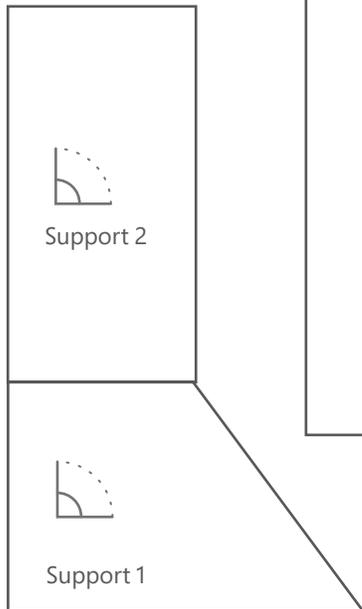
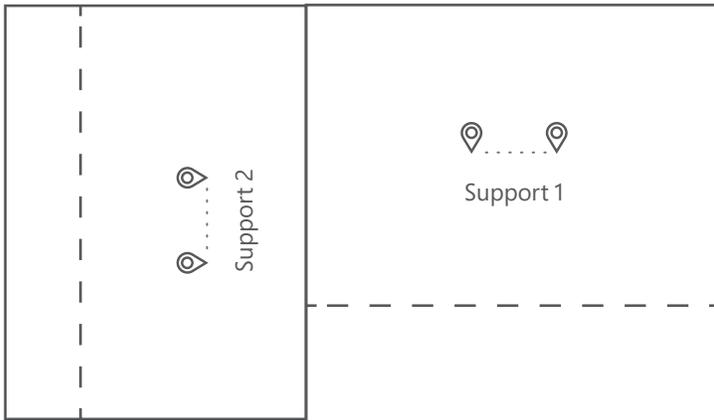
Continue to build and add elements until you've completed your island scene.











Print at 100%

— — — **Cut** solid line  
- - - **Fold** dashed line

