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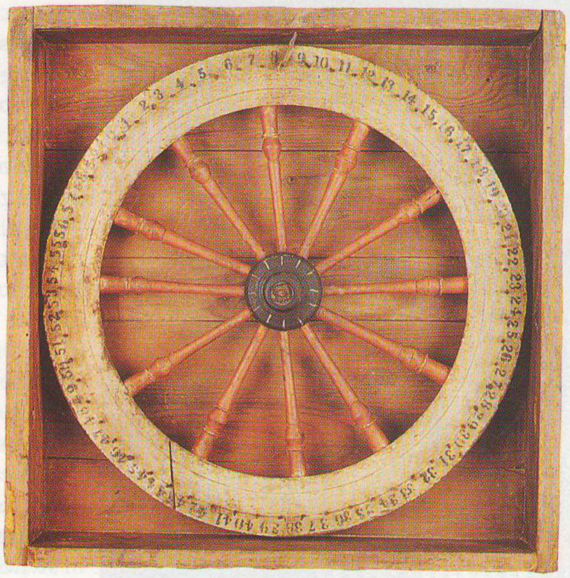
PLAYING BOTH SIDES

Parrondo's paradox shows that you can win at two losing games by switching between them. The result has surprising implications for the origins of life

BY ERICA KLARREICH

After a startling run-up, the stocks of Internet and technology companies have suffered a precipitous decline in the past year. A portfolio equally invested in such technology giants as America Online, Dell Computer, Lucent Technologies and Qualcomm, Inc., would have lost half its value between March 1 and December 1, 2000.

But investors can remain fully invested in the market and make money even if they own only poorly performing stocks—provided their portfolio includes several different losers. On the surface, it might seem that if owning one sinking stock were a bad idea, owning several would be insane. Mathematicians, physicists and biologists have proved otherwise.



Wheel of Fortune, c. 1900

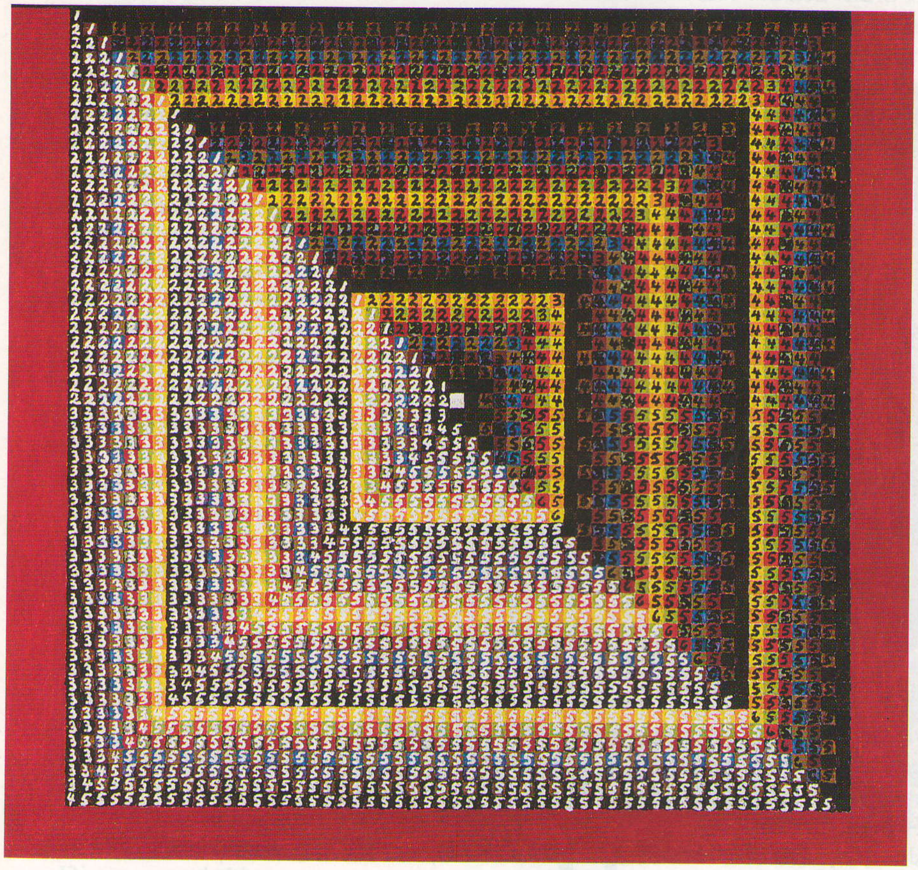
The key to such a counterintuitive investment strategy is to take advantage of random fluctuations in the prices of the stocks from day to day and from week to week. Even though all the general trends may be downward, the price of virtually any declining stock can also rally briefly if the market momentarily believes the stock is cheap enough to buy. If investors can manage to sell that stock during the brief price uptick that results from such market demand, and then shift those funds to another stock that is in a slump, they can overcome the general losing trends in both stocks. It is not an unproblematic strategy to pull off—transaction fees could gobble up profits, or a crash could drop prices monotonically for every stock in the portfolio. Yet the alternative, holding on to a losing stock as its value progressively slips away, offers no winning possibility at all, and could lead to far greater losses.

No one yet has found a foolproof way to play such odds profitably and reliably with a real losing stock portfolio. But it turns out, in a surprising number of circumstances, that it is quite possible to, in essence, "time the market." In other words, even when one is faced with a number of scenarios for which the odds of success are unfavorable—whether in games of chance, in choosing between competing political parties or in circumstances involving the second law of thermodynamics—it is possible to play two of the scenarios against each other to create a single winning strategy. The workings of that paradoxical result were elucidated recently by the Spanish physicist Juan M.R. Parrondo of Complutensian University in Madrid. Parrondo showed that what was once an obscure curiosity in physics could be relevant to daily life—and much more besides. Some investigators think Parrondo's paradox may hold the key to certain biological processes—and perhaps to the development of life itself.

Parrondo, as I have implied, was not the first to notice the paradox that bears his name. In 1992 the French physicists Armand Ajdari of the Paris School of Industrial Physics and Chemistry and Jacques Prost of the Curie Institute, also in Paris, discovered a microscopic engine that possesses an unusual property. If left on, the engine pushes particles to the left, but if turned on and off repeatedly, the engine moves particles to the right instead. The engine, which is now known as a flashing ratchet, operates by exploiting the random jiggling of particles known as Brownian motion.

In 1827 the English botanist Robert Brown observed that microscopic pollen grains suspended in water are in constant, chaotic motion. Those motions are the by-product of ceaseless random collisions with individual water molecules. Each individual particle—the invisible water molecules as well as the pollen grains visible in the microscope—acts in strict accordance with Newton's laws of mechanics. Yet taken together, the motions of the particles are so diverse as to be effectively random. Predicting the behavior of particles subject to the bumps and grinds of so many collisions is as frustrating and, ultimately, impossible, as predicting the outcome of a random coin toss.

Investigators frequently treat Brownian motion as noise, regarding it as an undesirable but unavoidable by-product of any machine that does useful work on the molecular scale. Engineers, however, have recently harnessed the very randomness of Brownian motion to generate orderly movement. Machines that can bring about such directed motions are known as Brownian ratchets, so named for the simple mechanical device that combines an asymmetric sawtooth wheel and a spring-loaded pawl, or rocker arm, to restrict rotary motion to one direction [*see illustration on opposite page*].



Alfred Jensen, According to the Numbers, Per. I, *1973*

In a mechanical ratchet, the pawl follows the contour of the toothed wheel. The teeth themselves are gently sloped on one side, and the pawl is mounted so that it slides smoothly over them when the wheel turns in one direction. On the other side of the teeth, however, the points fall away sharply, so that when a force tries to spin the wheel in the opposite direction, the pawl jams against a tooth, preventing the wheel from turning. Thus even when the ratchet is subject to forces that would move the wheel back and forth, the action of the pawl and the teeth allows the wheel to move in only one direction. Such asymmetry is also an essential part of the design of Brownian ratchets, including the one that inspired Parrondo's paradox.

To see how a group of objects in random motion can be ratcheted preferentially in one direction over another, imagine a collection of electrically charged particles distributed randomly along a gentle slope. Assume the particles are all in Brownian, random motion, which makes them jiggle with equal probability to the right and to the left along that line, with a slight drift leftward, say, down the slope. One can then superpose along the slope an electric potential that looks much like the teeth of a saw: periodic energy peaks rise steeply on the left side of the tooth and descend gently down its right side. (Engineers have built such potentials, for example, by depositing asymmetric electrodes along a line and then running a current through them.)

When current is turned off in such an apparatus, the charged particles simply jiggle in place along the line, with only a slight net downward drift. When the current is turned on, however, and the sawtooth potential is imposed, the particles are drawn toward the points of lowest energy, clumping together in the troughs of the potential.

Now switch the current off a second time, so that the particles resume their random Brownian jiggling. Since the troughs of the potential are offset to the right, as the particle drifts, it has a better chance of crossing the peak of the potential to its immediate right than the one to its immediate left. Switch the current on a second time, and most of the particles that drifted to the left will be swept back into the same trough from which they started. But some of the particles that drifted to the right will have drifted so far that they will be drawn to the next trough over. If such a potential is switched on and off, or "flashed," at the proper frequency, the charged particles in the flashing ratchet will march up the slope, overcoming gravity.

ALTHOUGH INVESTIGATORS could model the flashing ratchet mathematically, the mathematics for doing so is formidable. The complexity of the problem made it appealing only to a small coterie of physicists and mathematicians. "We've known for several years that there was this really counterintuitive phenomenon in physics," says Charles R. Doering, a mathematician at the University of Michigan in Ann Arbor, who coined the term "flashing ratchet" in the mid-1990s. "Parrondo's insight was to formulate the flashing ratchet in terms of games, which everyone can understand."

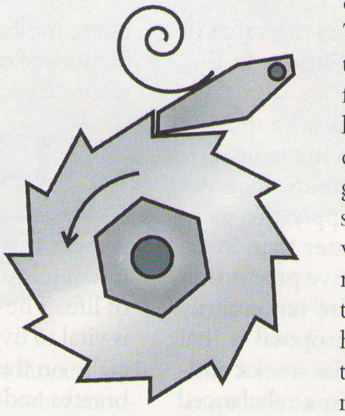
Since the motions of microscopic particles are as random as a coin toss, Parrondo knew he could translate the mechanics of the flashing ratchet into the language of game theory—and in the process reduce its mathematical content to simple fractions. "I thought that it would be really nice to explain something so surprising just with three simple numbers, such as 1/2, 3/4 and 1/10," says Parrondo. "I love simple things."

Parrondo invented two coin-tossing games to embody the essential elements of the flashing ratchet, and summoned his collaborator, the engineer Derek Abbott of the University of Adelaide in Australia, who was delighted with Parrondo's innovation. Together, they worked through the details over coffee, scribbling on napkins in a Madrid café. One coin-tossing game would be set up to mimic the Brownian motion of a particle on a slight incline when the potential of a flashing ratchet was turned off, providing a slow, steady drag on winnings. The second game would mimic the sawtooth bias of the flashing ratchet with the potential turned on. When he got back to Australia, Abbott and his colleague Gregory P. Harmer began computer simulations to see what would happen if they played the games a hundred times. Sure enough, when they played either one of the games for a long time, they lost steadily.

THE OUTCOME OF A SINGLE COIN TOSS IN either of the two games is identical: win the toss and gain a dollar, lose the toss and give back a dollar. The player begins with some random number of dollars. In the first game the player tosses a coin weighted so that the odds of winning are slightly less than 50 percent. In the second game two weighted coins are in play: the odds of winning with the first coin are nearly 75 percent in the player's favor; the odds of winning with the second coin are less than 10 percent. The player gets to toss the favorable coin whenever his pool of money is not a multiple of three. He must toss the unfavorable coin whenever his pool of money is a multiple of three.

It is obvious that the first game is a loser, but the long-term bias against the player in the second game takes a bit of explanation. Since the chances are two-to-one against beginning the game with a quantity of dollars that is a multiple of three, most players would begin with the coin weighted in their favor. Only in one start out of three would the player have to begin with the coin weighted heavily against him. So on the initial toss, the rough odds of winning are: $(2/3 \times 0.75) + (1/3 \times 0.1)$ or about 53 percent. Think of it as a "new customer bonus."

Play the second game long enough, however, and it becomes no longer true that the pool of money is divisible by three just a third of the time. A player whose pool of money is not a multiple of three tends to increase his earnings by a dollar with each coin toss, until the earnings reach a multiple of three. But any dollar amount divisible by three creates a large barrier to advancement—90 percent of the time a loss results. Hence after the first few plays the pool spends most of its time bouncing between amounts divisible by three and amounts just one dollar less. Although the random nature of the game means that for a single run of tosses the end result could be a tidy profit, play the second game enough times (Harmer and Abbott ran 50,000 simulations of 100 tosses each on their computer) and the trend shows up clearly as a gradual but inexorable loss.



Ratchet, pawl and spring

ONCE THE POOL GETS STUCK AT OR JUST below some multiple of three, it proves difficult to extract oneself simply by playing the second game. But Parrondo saw that switching to the first game was a way to jump out of that rut. Although the chances of raising one's earnings above a given multiple of three with two coin tosses are a measly 7 percent in the second game, the chances of doing so are nearly 25 percent in the first game. And once on the other side of that "energy barrier," switching back to the second game gives good odds of increasing the pot to the next multiple of three. Alternating between the two games—two coin tosses in the first game followed by two coin tosses in the second game—turns two losing propositions into one winning strategy. Again, the results of executing that strategy on any single run of tosses can vary wildly, but over the course of 50,000 runs of 100 tosses each, the simulations showed a trend for substantial gains. Remarkably, even when the switches between games were made at random, the overall gains were similar.

With Parrondo's strategy, a player can extract money from an opponent at no cost—"money for free," in the words of Abbott and Harmer. And with microscopic Brownian ratchets, there seems to be a similar magic at work. After all, the random motion of molecules in solution—analogous to the random outcomes of a coin flip—is translated into a steady flow in one direction. But there is a catch: the second law of thermodynamics. The second law states that a mechanical system cannot give rise to more order than is fed into it. In his discussion of the second law in his famous lecture series, the physicist Richard P. Feynman whimsically suggested that a microscopic ratchet-and-pawl system attached to a highly sensitive wind vane might draw enough energy from the random motion of air molecules in a room to winch up a flea. Feynman decided that his setup would violate the second law of thermodynamics—it would be a perpetual motion machine. The machine might indeed hoist the flea, he concluded, except that if the machine were really built on the molecular scale, the spring, too, would be subject to Brownian fluctuations, and so it would lift at random times. The ratchet could then turn in both directions.

At first glance, the flashing ratchet also appears to defy the second law. But a closer look shows that here there is an outside source of order acting on the system: the energy needed to switch the potential on and off. Brownian ratchets do not provide energy for free; like all motors, they merely convert one form of order, such as chemical or electromagnetic energy, into another form: directed motion.

But Brownian ratchets are unique in that noise is not a drawback for them but rather an essential ingredient. Many physicists had assumed that random motions would always hinder, not assist a machine; after all, random molecular motions are the essence of the disorderly state toward which the second law of thermodynamics inexorably pulls. But the flashing ratchet shows that such an assumption is unwarranted by the second law: the device would fail entirely if there were no Brownian motion to randomize the positions of the particles when the potential is turned off. "Noise is generally thought of as something undesirable, something to get rid of or to filter away," the biochemist Martin Bier of the University of Chicago has written. "To think of noise as something to exploit is a major shift of paradigm."

INTERESTINGLY, PEOPLE ALREADY EXPLOIT switching strategies similar to Parrondo's in everyday life—though not, of course, because they are applying theoretical insights about Brownian motion. But whenever someone changes jobs to get a salary increase, for instance, or sells peaking stocks and buys slumping ones, Parrondo's paradox comes into play. Moreover, many ordinary events are governed by random-seeming fluctuations, from the ups and downs of the financial markets to the succession of political parties in power. Economists and mathematicians have begun to ask whether the principles of Parrondo's paradox could apply to decision making in those realms.

CITIZENS COULD BE *following Parrondo's principles when they vote political parties alternately into and out of office.*

The physicist Sergei Maslov of Brookhaven National Laboratory in New York has shown that principles similar to Parrondo's paradox can be applied to the stock market. Maslov found a way that the value of a stock portfolio can increase even if all of its stocks decline in the long run: The entire portfolio must be sold periodically—even daily—and the proceeds must be quickly plowed back into repurchasing the stocks in the same proportions as they were in the original portfolio. The net effect of such periodic rebalancing is to apply the gains from stocks that are doing temporarily better than average (and have therefore increased their relative proportion in the portfolio) to buy more stocks that are temporarily underachieving. The beauty of Maslov's proposal is that one need not know anything about particular stocks; simply sell them all and then buy them back in a rebalanced portfolio. Unless the balance of investments is shifted in that way, the long-term prospects for a portfolio full of duds are "grim indeed," says Maslov, since all of the stocks tend to decline with time. But Maslov's tactic can generate better returns than the "buy and hold" policy that is the current favorite of many economists. (It should be noted, however, that the declines in technology stocks mentioned at the beginning of this article might well overcome even Maslov's strategy.)

Abbott and Parrondo suggest that switching from one option to another could also prove useful in applying government policies to quantities that fluctuate unpredictably; for example, if two policies each cause unemployment to rise, alternating between them might actually create jobs. Furthermore, Abbott notes, citizens could be, in essence, following Parrondo's principles when they vote political parties alternately into and out of office.

In short, Parrondo's paradox demonstrates that two wrongs can sometimes make a right—and that a little indecision can be a good thing. "One message of the Parrondian philosophy is to not be a moderate or an extremist, but to maintain a dynamic between positions that may appear to be conflicting," Abbott notes. "Parrondo gives us permission to be 'swing voters' in life's decisions."

As powerful as the insights of Parrondo's paradox may be at the macroscopic level, randomly switching from one "game" to another may have its greatest impact at the microscopic scale. In the past few years engineers working with such tools as lasers and microscopic electrodes have built ratchets to a degree of precision that would have been unimaginable just fifteen years ago. Such microscopic ratchets can work with objects of a truly molecular size—less than one-hundred-thousandth the size of Feynman's fleas.

The chemists Steven G. Boxer and Alexander van Oudenaarden of Stanford University have found that minuscule ratchets can sift through the jumble of proteins, lipids and carbohydrates that make up cell membranes. The two investigators observed that the ratchet allows particles to pass through it at different speeds, depending on their size and electrical charge. That property makes it possible to sort membrane molecules simply by passing them through a ratchet. Sorting membrane molecules could, in turn, become an important step in understanding just how particular drugs attach themselves to receptors in cell membranes.

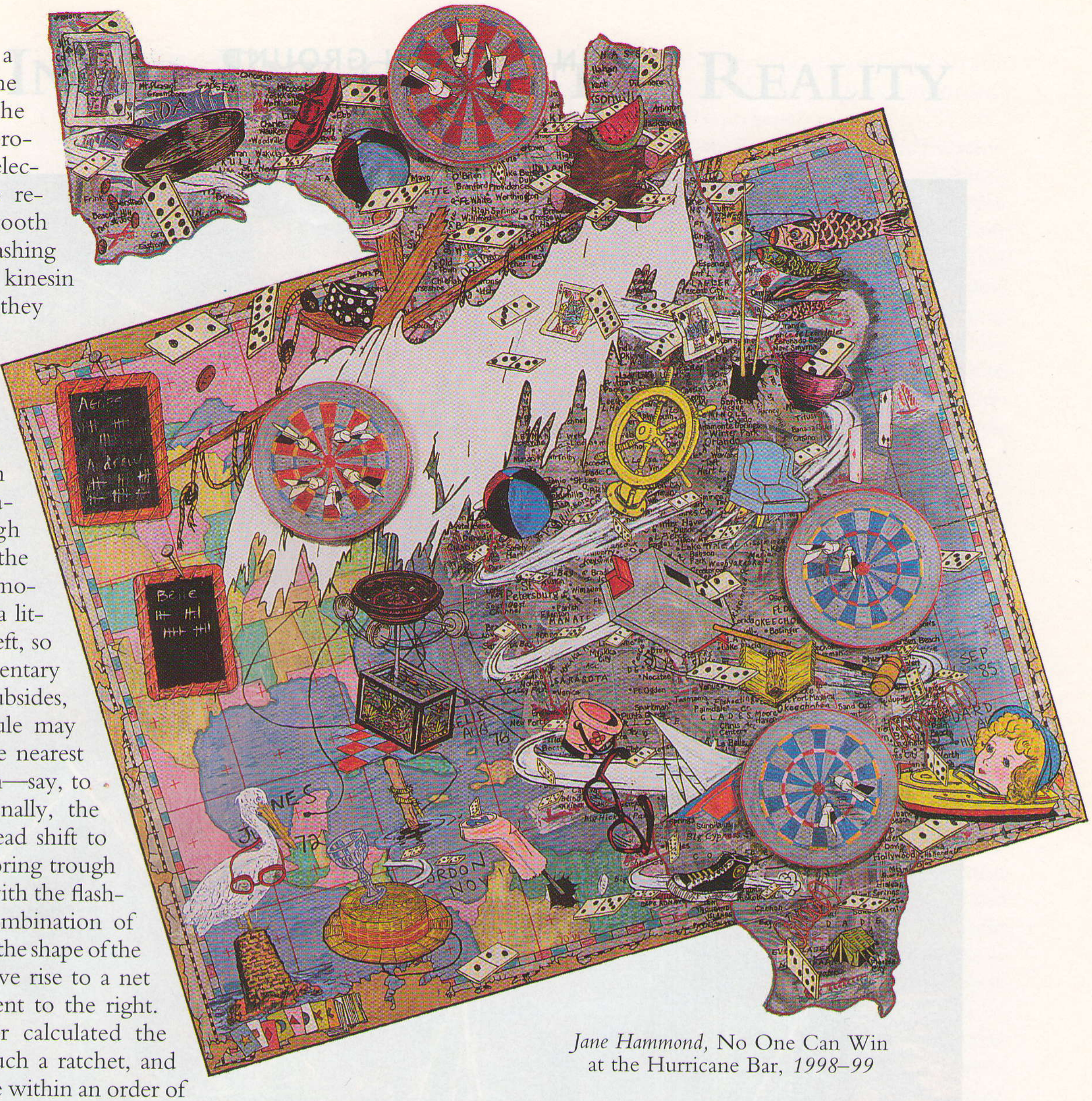
EVEN AS ENGINEERS ARE DEVELOPING THE technology to build and use minuscule Brownian motors, biologists are beginning to suspect that nature has been exploiting the properties of intricately designed Brownian ratchets since the beginnings of life. The ability to convert energy into directed motion is vital to living creatures; people, for instance, depend crucially on the muscular contractions that pump blood, draw breaths and move arms and legs. On a much smaller scale, cellular motors perform such chores as transporting organelles. The fuel for those motors is well known—the chemical energy stored in the molecule adenosine triphosphate, or ATP. But the inner workings of the motors themselves remain enigmatic.

Some investigators have proposed recently that so-called motor proteins, which haul microscopic cargo, might operate in part by a ratchet mechanism. In 1993 a team of engineers at Harvard University and the Rowland Institute for Science in Cambridge, Massachusetts, tracked the movements of the motor protein kinesin. That protein moves by propelling itself along cellular "train tracks" called microtubules, and can transport cargo more than a hundred times its size—much like an ant carrying an enormous cake crumb. The team discovered that instead of moving smoothly along the microtubule, kinesin makes distinct jumps, each about eight nanometers long, as if it were climbing the rungs of a ladder.

Those discrete jumps led Martin Bier and Dean Astumian, a colleague of Bier's at the University of Chicago, to hypothesize that a ratchet-like engine was at work. The microtubule, they proposed, could be electrically charged to resemble the sawtooth potential of the flashing ratchet. When the kinesin molecule is at rest, they suggested, it prefers to stay in one of the troughs of the potential. But when ATP is burned, the kinesin molecule temporarily gains enough energy to escape the trough. Brownian motion makes it drift a little to the right or left, so that when the momentary burst of energy subsides, the kinesin molecule may slide down into the nearest neighboring trough—say, to the right. Occasionally, the molecule may instead shift to the nearest neighboring trough on the left, but as with the flashing ratchet, the combination of jiggling motion and the shape of the sawtooth should give rise to a net long-term movement to the right. Astumian and Bier calculated the power output of such a ratchet, and their estimates came within an order of magnitude of the measurements made by the Harvard and Rowland Institute team. Since that time, several other models have been suggested that fit even more closely with experimental observation. Each model exploits a Brownian ratchet engine, along with other mechanisms.

Engineers hope that the mechanism of kinesin action, when finally understood, will provide the blueprints for minute, man-made motors that could be useful in microelectronic devices. "Ion pumps and motor proteins are the smallest engines we know and they can no longer be thought of as wind-up toys," Bier has written. "If nano-technological devices are to be as efficient and reliable as biological molecules a good understanding of noise will be indispensable to their design."

BEYOND EXPLAINING THE INNER WORKINGS OF living creatures, some investigators suggest that Parrondo's paradox may hold the key to explaining one of the oldest puzzles of nature: how life began. One central problem is to explain how the inchoate, randomly jiggling mass of particles that physicists call primordial soup could give rise to the complex molecules of life.



Jane Hammond, No One Can Win at the Hurricane Bar, *1998–99*

But Parrondo's paradox demonstrates that processes leading to net losses can be combined to generate a positive outcome, and that randomness can facilitate that outcome. Abbott suggests that a mechanism such as the flashing ratchet might have harnessed forces that would otherwise have led to disorder, and channeled them toward the assembly of complex amino acids. Put in a slightly different way, processes that would, if left to themselves, tear up molecules essential to the chemistry of life, might have worked together instead to foster the synthesis of such molecules.

Thus perhaps the emergence of order is not as unlikely as it has often seemed. The physicist Paul C.W. Davies, who examined the question of how life began in his book *The Fifth Miracle,* has observed:

> Nobody knows what the process was, but anything that clarifies how information and organized complexity can emerge from the randomness of molecular chaos will cast welcome light on this most profound of scientific mysteries. That is the significance of Parrondo's paradox, and the associated work of people like Derek Abbott. •

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