THE SHAPE OF SPACE

Have cosmologists glimpsed signs that the universe is bounded?

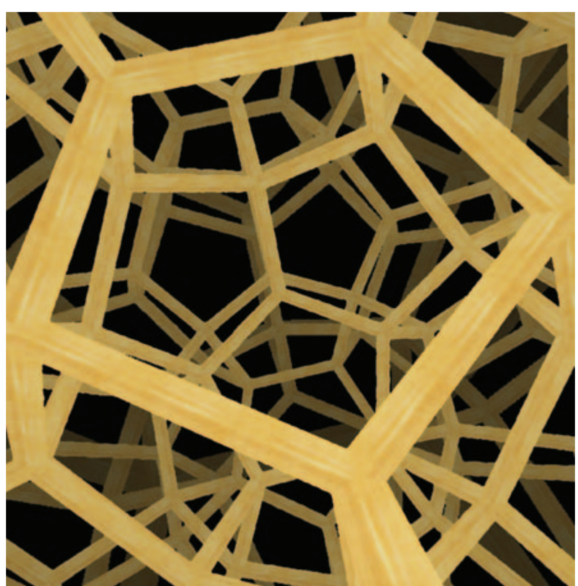
BY ERICA KLARREICH

Gaze deep into the night sky, and space appears to extend infinitely far in all directions. Given such a view, it's mind-boggling to think that space might be bounded. Yet, just as the flat-seeming Earth is in fact a sphere, infinite-seeming space may curve in on itself to close up into a compact shape. Recently, the debate over the shape of space took some new twists. In the Oct. 9 *Nature*, a team of mathematicians and astrophysicists proposed an exciting idea. The universe may have a particular finite shape, modeled on a 12-sided geometric object known as a dodecahedron, they propose. The same week, a second group of scientists announced findings that may refute that proposal.

Both groups have based their analyses on first-year data from NASA's Wilkinson Microwave Anisotropy Probe (WMAP), which in February produced a snapshot of temperature waves shortly after the Big Bang (*SN: 2/15/03, p. 99*). These waves produced a puzzle: One of the longest wavelengths, known as the quadrupole, is less powerful than expected. This is like saying, in an analogy with sound waves, that the universe doesn't play low notes.

To many cosmologists, the reduced quadrupole is a hint that the universe may be finite. In an infinite universe, all wavelengths should be equally abundant, whereas in a finite universe, waves can never be longer than the universe itself.

By analogy, "you don't get really long waves in a bathtub because the waves can't be bigger than the bathtub is long," says Jeffrey Weeks, a freelance geometer based in Canton, N.Y., who is one of the authors of the *Nature* paper.



HALL OF MIRRORS — In the dodecahedral model of the universe, each pentagonal face is theoretically glued to the opposite face. In such a universe, if you look straight through one of the faces, you see what looks like a new dodecahedron but is really the original dodecahedron seen from a new vantage point.

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COMPLEX SOCCER BALL Weeks and his coauthors report that a shape called the Poincaré dodecahedral space is a good fit for both the quadrupole data and estimates of the universe's curvature.

The Poincaré dodecahedral space is formed by gluing together opposite faces of a slightly curved dodecahedron—a soccer-ball-like shape with 12 pentagonal sides. Such a gluing is impossible to carry out physically within ordinary three-dimensional space. However, by keeping track of which faces are theoretically glued, scientists can measure the physical attributes of such a space.

If the universe had this shape, a traveler who crossed through one of the pentagonal faces would instantly reappear at a face on the opposite side of the dodecahedron. Video game characters make such treks in two dimensions when they vanish from one side of the screen and reappear at the other. In the dodecahedral universe, a trip across the solid would span many billions of light-years.

Weeks and his collaborators were drawn to study the dodecahedron because recent observations of the universe's cosmic microwave background radiation have suggested that the universe either is flat or has slightly positive curvature, such as a sphere does. This estimate makes many of the possible shapes for the universe unlikely. The few more-likely candidates include the dodecahedron and a shape called the three-torus, made by gluing opposite sides of a box.

Preliminary examinations of various kinds of three-torus—made from boxes of different shapes and sizes—have yielded no shape that fits the quadrupole data well. The dodecahedron model, however, appears to match data on both the quadrupole and the next-longest wavelength, called the octopole.

NOT SO FAST The Poincaré dodecahedron's apparent match with the quadrupole data and the curvature measurement is "intriguing," says David Spergel, an astrophysicist at Princeton University and a member of the WMAP team. However, at a cosmology conference in Cleveland on the day after the *Nature* paper appeared, Spergel reported findings that he says undermine the dodecahedron model.

Spergel and his collaborators examined whether the dodecahedron satisfies a criterion called the circle test. This criterion rests on the observation that, because we can see equally far in all directions, the boundary of our visible universe is an enormous sphere.

Think of our visible universe as a bubble at the center of the dodecahedral model. If the bubble were much smaller than the dodecahedron, we would see nothing of the pentagonal faces. If the bubble were to grow, it would eventually touch each pentagonal face at a single point at the center. If the bubble were to grow a tiny bit further, it would cross each face, intersecting it in a circle.

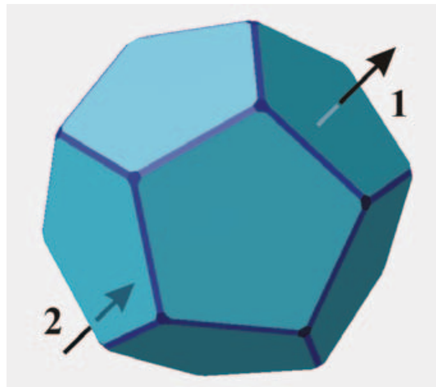
According to the model in the *Nature* paper, the dodecahedral universe is just the right size for the bubble to intersect the pentagonal faces in circles. Because each of the 12 pentagonal faces is glued to the opposite face, each circle should match an identical circle on the opposite pentagon. Therefore, the sky should contain six pairs of matching circles on which all the physical data, including the temperature waves in the WMAP data, are identical.

Spergel and his collaborators have been combing the WMAP data looking for such circle pairs. According to Spergel, the circles are not there.

Weeks agrees that the absence of matching circles would kill the dodecahedron model. However, he says, it's possible that Spergel's team missed the circles. Although in principle we should see identical light coming from the matching circles in the sky, in reality, effects such as noise obscure the circle pairs. If the noise is strong enough, Weeks says, the circles might elude detection by the algorithm Spergel's team used.

The WMAP team has tested its algorithm on a simulated sky map that factored in noise and other distortions, says Neil Cornish of Montana State University in Bozeman, one of Spergel's collaborators. According to those simulations, if circles are in the sky, there is less than a 1 percent chance that the algorithm would miss them.

However, the team carried out its test simulations in a three-torus model rather than a dodecahedral model. It's not clear whether the 1-percent-error estimate would carry over to the dodecahedron, Weeks says.



SPACE TREK — In dodecahedral space, if you journey across one pentagonal face (1), you instantly reappear through the opposite face (2), on the other side of the universe.

WEEKS

The circle search team plans to run its simulations on the dodecahedral shape. "We're going to be able to make extremely strong statements about their model," Cornish says. "It's such a dramatic claim that it's worth going that extra mile to test it."

However, he does not expect the simulations to validate the dodecahedron model. "I really think there isn't any room left," he says.

TESTING, TESTING Until Spergel, Cornish, and their collaborators have performed simulations on the dodecahedron and cosmologists have had a chance to scrutinize the work, it's premature to agree or disagree with the team's findings, Weeks says. "The recent history of cosmic topology is littered with examples of people claiming to rule out various possibilities, only to find later that their analysis had flaws," he notes. At this point, he says, both the dodecahedron model and the evidence against it should be considered with caution.

If the dodecahedron is ruled out, "it will be a disappointment," Weeks says. "It fits the data really well, and there aren't a lot of backup candidates to go." Among the plausible shapes that remain to be tested is a three-torus made out of a slanted box.

In an ambitious project, Spergel, Cornish, and their colleagues are scanning the sky for any evidence of circle pairs that would indicate these or other possible shapes. That wider search should be completed soon, Cornish says.

"What's nice is that we don't need to launch another satellite to test [the models]," says Max Tegmark, a cosmologist at the University of Pennsylvania in Philadelphia. "The answer is there in the data and just needs to be ferreted out."

If the dodecahedron model turns out to be incorrect, the question of why the universe doesn't play low notes will remain.

"We don't know whether it's a fluke, a compact universe, or some other cosmological effect," says Charles Bennett of NASA's Goddard Space Flight Center in Greenbelt, Md., who is the director of the WMAP project. "That's the exciting part." ■